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A TROUGH AS WAVEGUIDE FOR WHISTLERS IN THE MAGNETOSPHERE

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SUMMARY: Ray tracing calculations show that the existence of the trough in the magnetosphere is capable to guide whistlers from one hemisphere to the other. The guidance of whistlers occurs only around or by the trough's inner side, when a trough is located at L $\leq$ 2.2 and L $\geq$ 4.0 in the winter night model and at L $\geq$ 1.5 in the summer day model of the magnetosphere. Initial latitude ranges for various wave normal directions, are moved equatorward from the trough center. For one trough the initial latitude ranges are defined so that corresponding rays reach 300 km altitude in the conjugate hemisphere with a final wave normal angle inside the ionospheric transmission cone. These whistlers are characterized by a nose frequency and an upper cut-off frequency which is $\sim$ 0.5 f_{Bmin}.

1. INTRODUCTION

The phenomenon of whistling atmospherics or whistlers is a part of the radio emissions from lightning, which is dispersed by magnetized plasma in the ionosphere and magnetosphere. The whistler mode is defined as radio waves propagating in the frequency range lower than the plasma frequency and the gyrofrequency of the medium. The first ray theory of whistler propagation that is supported by field oriented irregularity of the electron density was presented by Smith *et al.* (1960). They showed that the trapping was possible in both enhancements and depressions of the ionization. Many authors have concluded that whistlers observed on ground, propagated in ducts of enhanced electron density (for example Smith, 1961; Helliwell, 1965; Strangeways 1991; Laird 1992). Most of whistlers observed in the ionosphere and magnetosphere by scientific satellites are nonducted mode.

It has been shown that under reasonable initial conditions an electric field in the equatorial plane can produce electron density enhancements and depressions in the magnetosphere. This mechanism does not add or subtract plasma from the magnetosphere, but merely strips up the existing plasma, an enhancement can be produced only at the expense of a depression somewhere else (Park and Helliwell, 1971; Lester and Smith, 1980).

In this paper we have shown how the trough can act so as to guide whistler mode waves along the geomagnetic field lines, in a manner similar to normal ducting. Wave guidance in the magnetosphere by one-side irregularities has been considered by Voge (1961; 1962) and Gorney and Thorne (1980).

The question of plasmapause guidance of VLF waves has also been addressed in the literature. Inan and Bell (1977) have shown that the plasmapause negative density gradients can guide VLF waves. Gradient trapping of low and high frequencies by the trough existence in the magnetosphere has been analyzed by Helliwell (1965).

To solve the problem of defining initial positions of multicomponent whistlers Šulić and Grubor (1996) have supposed the existence of troughs in the high latitude magnetosphere and shown the guidance of whistlers by them.

This paper reports ray tracing results about the guidance of whistlers by a trough incorporated in the winter night (WN) and summer day (SD) model of the magnetosphere. The attention is paid to the guidance of whistlers which reach 300 km altitude in the conjugate hemisphere with a final wave normal angle inside the ionospheric transmission cone.

2. ELECTRON DENSITY PROFILE IN THE MAGNETOSPHERE

In the magnetosphere the electron and ion densities are represented by a field aligned isothermal diffusive equilibrium model (Angerami and Thomas, 1964). The electron density at any point in the magnetosphere is calculated from the electron, ion (H^+, He^+ and O^+) and temperature profiles at a reference level of 900 km. The charged particle temperature and densities at the reference level are given by polynomial and exponential fits to satellite data for different seasonal and diurnal conditions (Denby *et al.* 1980). The results presented in this paper are obtained by ray tracing calculations for cold plasma.

The refractive index is calculated for a cold collisionless plasma composed of electrons and three species of ions.

$$\mu^2 = 1 + \frac{f_p^2}{f(f_B \cos\Psi - f)} \qquad (1)$$

where f_p is the plasma frequency, f is the wave frequency and f_B is the electron gyrofrequency. The angle between wave normal and the geomagnetic field is denoted by Ψ. As it is known f_p^2 is proportional to electron density and f_B to the geomagnetic field strength (Helliwell, 1965). The refractive index has value larger than unit and especilly very large when the plasma frequency and the electron gyrofrequecy are much larger than the wave frequency. The refractive index is greatly dependent on the direction of propagation with respect to the direction of geomagnetic field. In this paper the geomagnetic field is assumed to be a dipole field.

Plasma diffuses along field lines rather than across them, so enhancement or depression in plasma density tends to be field-aligned (Smith, 1961; Helliwell, 1965; Strangeways, 1991; Laird, 1992). A duct or trough with a Gaussian cross-section can be incorporated into symmetric model of the magnetosphere, which gives an additional enhancement or depression in electron density, respectively. Namely,

$$N_e = N_{eo}[1 \pm \delta \exp(-x^2/2\sigma_d^2)] \qquad (2)$$

where N_{eo} denotes the slowly varying background electron density; x is the separation (in km) between the field lines corresponding to the ray position and that of the trough center; δ is electron density enhancement at the duct center or depression at the trough center and σ_d is the distance either side of the duct or trough center in the equatorial plane where the electron density has the value of $N_{eo}[1 \pm \delta/\sqrt{e}]$. An enhancement or depression of the electron density has the value of one-tenth of δ at $2.15\sigma_d$ either side of the duct or trough center, respectively (Angerami, 1970; Strangeways, 1991).

In our ray tracing calculations a Gaussian cross-section trough has been modeled with different electron density depression and effective width in the equatorial plane. The full depression at a trough center was reached at 2100 km altitude in the WN model and at 1000 km in the SD model. From those altitudes the central depression at $x = 0$ varied linearly from given values to zero at 300 km altitude.

3. THE INFLUENCE OF THE MAGNETOSPHERIC MODEL ON THE EFFICIENCY OF TROUGHS

To analyze the influence of the density model to the trough characteristics in guiding VLF waves, ray tracing calculations for the WN and SD model of the magnetosphere have been done by Šulić (1996). General results are:

- The existence of trough ($\delta = -15\%$ and $\sigma_d = 50$ km) can guide successfully whistlers when it is located at L $\leq$ 2.2 and L $\geq$ 4 in the WN model. A trough incorporated at any location in the SD model for L $\geq$ 1.5 enables guiding of VLF waves effectively from one hemisphere to the other. L value is the geocentric distance in the equatorial plane divided by Earth's radius. These whistlers can penetrate the ionosphere and might be observed on the ground.
- A guidance occurs around or by a trough's inner side.
- Initial positions are located equatorward from the trough center.

Figure 1 shows meridian plane cross-section of the magnetosphere, with ray paths for 3 kHz rays. Rays started at the southern hemisphere and were guided by the troughs located at L = 2.7 and L = 4.7 in SD model.

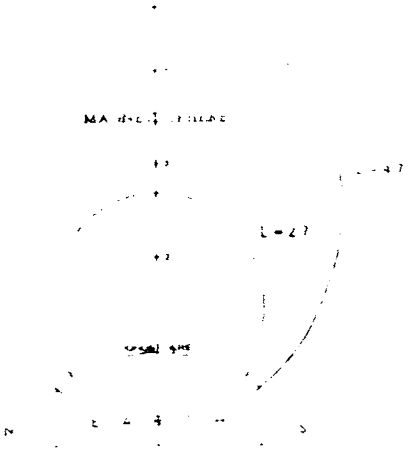

Fig. 1. Illustration of the ray paths for 3 kHz rays guided by troughs located in SD model.

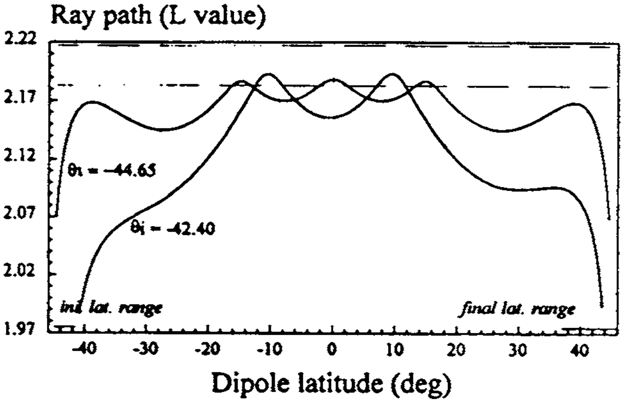

Fig. 2. Ray path parameters for 5 kHz rays starting at different initial latitudes in the WN model.

To present the guidance of whistlers in the WN model of the magnetosphere we have selected example of 5 kHz rays guided by trough at L = 2.2 (δ = -15% and σ_d = 50 km). Figure 2 shows ray paths versus dipole latitude for 5 kHz rays injected with initially vertical wave normals at different initial latitudes from 300 km altitude. Two dashed lines represent inner and outer side of the trough, where electron density depression has one-tenth values of δ. Dotted line represents the projection of the equatorial plane.

After the ray leaves initial position, the wave normal is moved away from the vertical towards the geomagnetic field direction. This movement is effected by latitudinal gradients of electron density and geomagnetic field in the low altitude magnetosphere. These effects are sufficient to bring the wave normal and ray defection into coincidence with geomagnetic field direction.

The refractive index has a minimum value at 1930 km altitude in the starting hemisphere and then increases with altitude reaching maximum value in the equatorial region. As a ray approaches the region of minimum refractive index, it is refracted inward. There the curvature of the magnetic field plays dominant role and deflects a ray outward. A ray enters the trough through inner side. An additional negative radial electron density gradient acts so as to refract a ray inward and the process is repeated. The ray reaches 300 km altitude in the conjugate hemisphere with a final wave normal angle inside the ionospheric transmission cone.

Our results show that a ray can be trapped by negative density gradient around the trough's inner side. Its wave normal oscillates around the direction of the geomagnetic field that corresponds to the trough's inner side. This whistler has the nose frequency (fn) of $\sim$ 31.5 kHz and the upper cut-off frequency (fuco) of $\sim$ 40 kHz, which corresponds to one half of the minimum value of gyrofrequency (f_{Bmin}) along the ray path.

The propagation of these rays at low altitudes in both hemispheres is well outside the influence of the trough and it is *a function of the ambiantal field and density model.* We have performed ray tracing calculations for troughs located at the same L value in both models, in order to check how sensitive the results of initial and final latitudes and final wave normal angles are to the choice of model.

Here are presented results for troughs located at L = 4.2 in WN and SD model of the magnetosphere. Figure 3 shows initial latitude range versus initial wave normal angle to the vertical for 3 kHz ray started at 300 km altitude in the southern hemisphere. The scale of the axes of the initial latitude starts with dipole latitude of trough center. At 300 km altitude it is at 60.05°S. For SD model rays with wave normal directions inside the cone from -9° to 23° are successfully interhemisphric guided. For WN model this cone is from -2° to 16°. The particular initial latitude range for the same direction of the wave normal is much wider for SD model than for WN model. It is interesting to point out that all particular initial ranges are moved equatorward from the inner side of the trough.

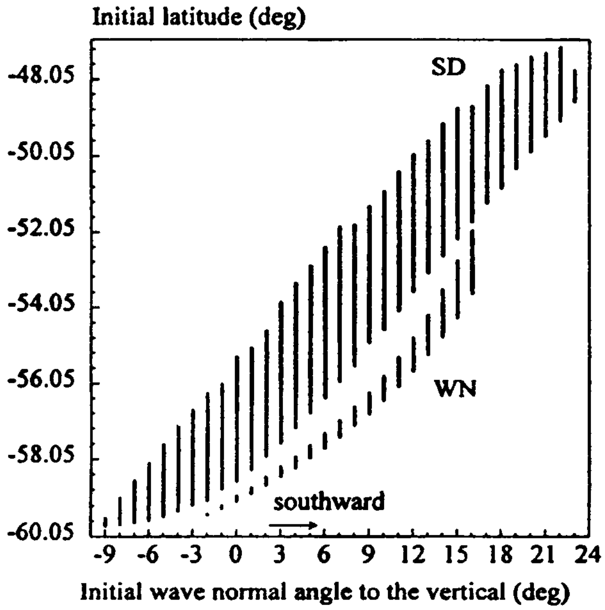

Fig. 3. Initial latitude ranges versus initial wave normal angle to the vertical for 3 kHz ray guided by a trough at L = 4.2.

SD model is characterized by larger electron density of the ambiental plasma than for WN model. Latitudinal gradient of the electron density at the lower altitude magnetosphere for SD model is more efficient to bring the wave normal and ray direction into coincidence with geomagnetic field direction.

4. RAY PATH DEPENDENCE ON THE WAVE FREQUENCY

Whistler ray paths in the magnetosphere are primarily influenced by plasma density gradients and the geometrical effects of field-line curvature. Variations in the magnitude of the ambient magnetic field, or the ratio f/f_{Bmin}, play a relatively minor role.

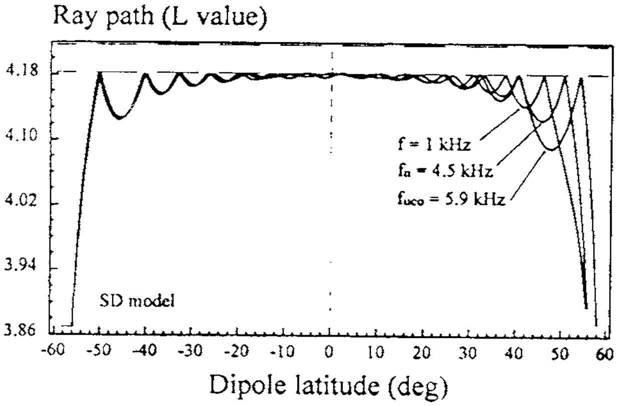

Fig. 4. Variations of ray paths for different frequencies, started at the same initial position in SD model.

Figure 4 shows ray paths for three frequencies: f = 1; 4.5 and 5.9 kHz versus dipole latitude. All rays started with initially vertical wave normals in SD model at 300 km and at θi = 57.55°S. The results show that all rays follow very similar shape of ray trajectories.

Ray paths for frequencies f = 2.8; 4.5 and 5.8 kHz versus dipole latitude are presented in Figure 5. These rays were guided by the trough centered at L = 4.2 in the WN model. Rays started with wave normal angle i = 1° at 300 km altitude and at θi = 58.92°S. The guidance of these rays is carried out by very similar shape of ray trajectories. The rays reach 300 km altitude in the conjugate hemisphere with final wave normal angle inside the ionospheric transmission cone.

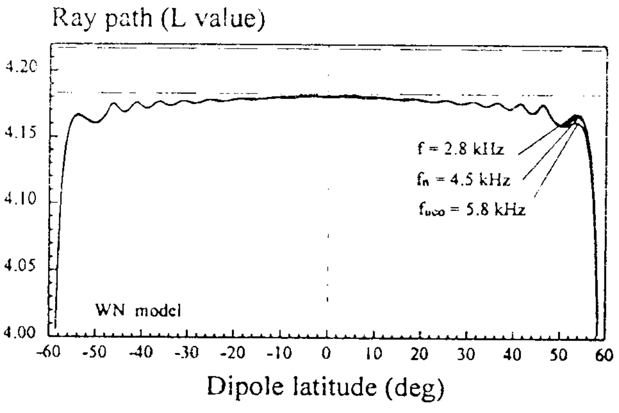

Fig. 5. Variations of ray paths for different frequencies started at same initial position in WN model.

These results show that all frequencies of whistler guided by a trough, follow very similar shape of trajectories in two models of the magnetosphere.

5. THE INFLUENCE OF TROUGH'S WIDTH ON GUIDING WHISTLERS

To analyze this problem we have supposed the existence of troughs at L = 2.7 in the SD model, with widths in the equatorial plane of 43 km; 215 km and 430 km. The electron density depression was δ = -15% in all of them.

Ray tracing calculations have been done for 5 kHz rays with initially vertical wave normals at 300 km altitude. The main results are:

- For a trough with a width of 43 km initial latitudinal range is about 05° and it is moved equatorward. Final latitude range is from 47.65°N to 49.80°N ($\Delta\theta f$ = 2.15°). Most of them have final wave normal angle inside the ionospheric transmission cone.

- For a trough with a width of 215 km the initial latitudes are spread from 50.15°S to 48.35°S ($\Delta\theta i$ = 1.8°). The final latitude is from 41.30°N to 49.60°N ($\Delta\theta f$ = 8.3°).

- For a trough with a width of 430 km the initial latitude is spread from 50.0°S to 48.30°S ($\Delta\theta i$ = 1.8°). The final latitude range is from 42.10°N to 49.40°N ($\Delta\theta f$ = 7.3°).

A narrow trough has a steep negative gradient of electron density, which causes a very narrow initial latitude range. From this initial latitude range practically all injected rays reach 300 km altitude in the opposite hemisphere with final wave normal angle inside the ionospheric transmission cone. Wave normal and ray direction are brought into coincidence with the geomagnetic field line, which corresponds to the trough's inner side.

6. THE ANALYSIS OF TROUGH PROFILES IN GUIDING WHISTLERS

As the first approximation for field-aligned irregularities in the magnetosphere, we have supposed the existence of a Gaussian cross-section or a quartic trough in different models of the magnetosphere. Here are presented results for these two types of troughs located at L = 2.7 in SD model. These troughs were modeled with density depression of -15% and a width in the equatorial plane of 215 km.

Figure 6 presents ray tracing calculations of ray paths versus dipole latitude. 5 kHz rays started with initially vertical wave normal directions at 300 km altitude and at θi = 50.0°S, in SD model. The curve denoted by a corresponds to ray guided by **a** quartic trough and the curve denoted by **b** to a ray guided by a Gaussian cross-section trough.

The initial latitude range for Gaussian cross-section trough is wider than for quartic trough. This is a consequence of steeper negative gradient of the electron density inside a quartic trough than in a

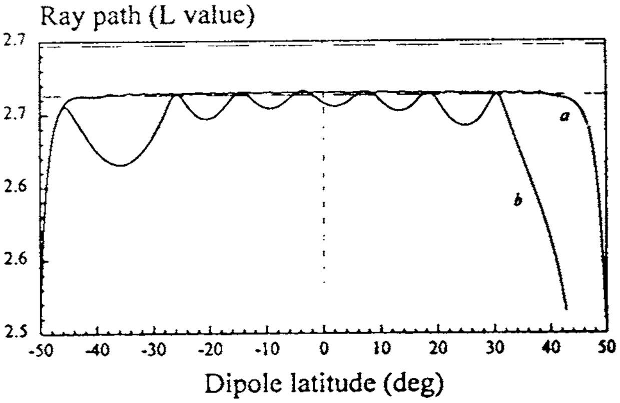

Fig. 6. Ray path parameters for 5 kHz rays guided by a quartic and a Gaussian cross-section trough.

Gaussian trough. Also this very steep negative gradient of the electron density in a quartic trough, defines a vary narrow band around trough's inner side in which rays are guided. It is interesting to point out that in this case the wave normal angle to the geomagnetic has values of $\Psi \sim |1°|$.

7. CONCLUSION

We have carried out ray tracing calculations to show that the existence of trough is capable of guiding whistlers from hemisphere to hemisphere. The main results from the present study are summarized in the following.

1. For realistic model of the electron density of the ambiental plasma and field model *the upper cut-off frequency for whistlers guided by a trough is fuco* $\sim$ *0.5* f_{Bmin}. Whistlers exhibit all properties of signal propagated by normal duct. Namely, whistlers can penetrate the lower ionosphere, and might be observed on the ground.

2. A trough incorporated in the middle latitude at L $\leq$ 2.2 and in the high latitude at L $\geq$ 4.0 WN model and at L $\geq$ 1.5 in SD model of the magnetosphere can guide whistlers from hemisphere to hemisphere.

3. Initial and final latitudes are spread equatorward from the trough center. This is a consequence of guiding whistlers around or by the trough's inner side.

4. Narrow troughs and troughs with very steep radial gradient of the electron density have vary narrow initial latitude range. These troughs guide whistlers with wave normal and ray direction practically parallel to the geomagnetic field direction, which corresponds to the trough's inner side.

5. Very wide troughs are capable to guide interhemispherically whistlers, which cover wide initial latitude range. These rays reach 300 km altitude in the conjugate hemisphere with final wave normal angles spread in a large cone.

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УВАЛА У МАГНЕТОСФЕРИ КАО ТАЛАСОВОД ЗА ТАЛАСЕ У МОДУ ЗВИЖДУКА

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Прорачуни "ray tracing" методом показују да постојање увале у магнетосфери (смањење густине електрона у радијалном правцу у односу на околину) омогућава простирање таласа у моду звиждука од једне хемисфере до друге. Ово простирање звиждука одиграва се око или дуж унутрашње стране увале, када је увала лоцирана на L $\leq$ 2.2 и L $\geq$ 4 у моделу магнетосфере за услове зимске ноћи и када је лоцирана на L $\geq$ 1.5 за услове летњег дана. Вредност L се одређује тако што се геоцентрична удаљеност у екваторијалној равни подели са полупречником Земље. Опсези диполних ширина са којих се екситују таласи (са таласним нормалама у различитим правцима) смештени су између екватора и увале. За једну увалу иницијални опсег диполне ширине одређује се тако, да таласи који се са њега екситују, доспевају до 300 km висине на супротној хемисфери. Главна особина ових таласа је да продиру кроз јоносферу и могу бити регистровани на површини Земље. Таласи се простиру у моду звиждука и имају носну фреквенцију (fn) која износи (0.37 f_{Bmin}. То је фреквенција која има минимално време путовања од једне хемисфере до друге. Горња прекидна фреквенција (fuco) је горња граница фреквентног опсега звиждука, који се простире дуж увале, и износи $\sim 0.5\ f_{Bmin}$.