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A NEW APPROACH TO THE THIELE–INNES ELEMENTS CALCULATION

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SUMMARY: Bearing in mind the geometrical meaning of the Thiele–Innes elements A, B, F and G the analytical expressions for these constants as functions of the axes and of the apparent-ellipse-positions parameters of a double star are given. This procedure has been successfully applied to several double stars. The orbital elements obtained in this way can be used as the initial ones in the application of Eichhorn's procedure (Eichhorn, Xu, 1990).

1. INTRODUCTION

At the Belgrade Observatory a significant number of orbital- elements sets for visual double stars has been obtained by using the geometric-analytical procedure. The Innes constants A, B, F and G are determined by applying the geometrical procedure and the next step is to obtain the orbital elements from their values. The Innes constants are taken from the plots knowing their geometrical meaning. The development of the computing technique has made it possible to introduce a new, more convenient and more efficient, procedure for the computation of constants A, B, F and G instead of the geometrical one. Here such a procedure is proposed.

After being reduced to the same epoch, the observations are transmitted to the computer monitor and in the following step the apparent binary-system orbit is realized by choosing its dimensions, position and direction in agreement with the observed ellipse arc. The drawing of the apparent ellipse on the monitor is done by defining the ellipse axes (a, b), the translation parameters (m, n) and the rotation (α) of the coordinate system with the main component of the system at the coordinate origin. The Innes constants can be then defined as functions of these parameters, namely

$$\begin{aligned} A &= f_1(m,n,a,b,\alpha) \qquad & F &= f_3(m,n,a,b,\alpha) \\ B &= f_2(m,n,a,b,\alpha) \qquad & G &= f_4(m,n,a,b,\alpha) \end{aligned} \tag{1}$$

Further procedure of geometrical-orbital-elements determination is carried out with the known formulae connecting the Innes and the Campbell constants (Heintz, 1972).

The dynamical elements of the orbit – period (P) and periastron-passage time (T) – are obtained here by using the least-square method from the linear dependence mean anomaly-observation time.

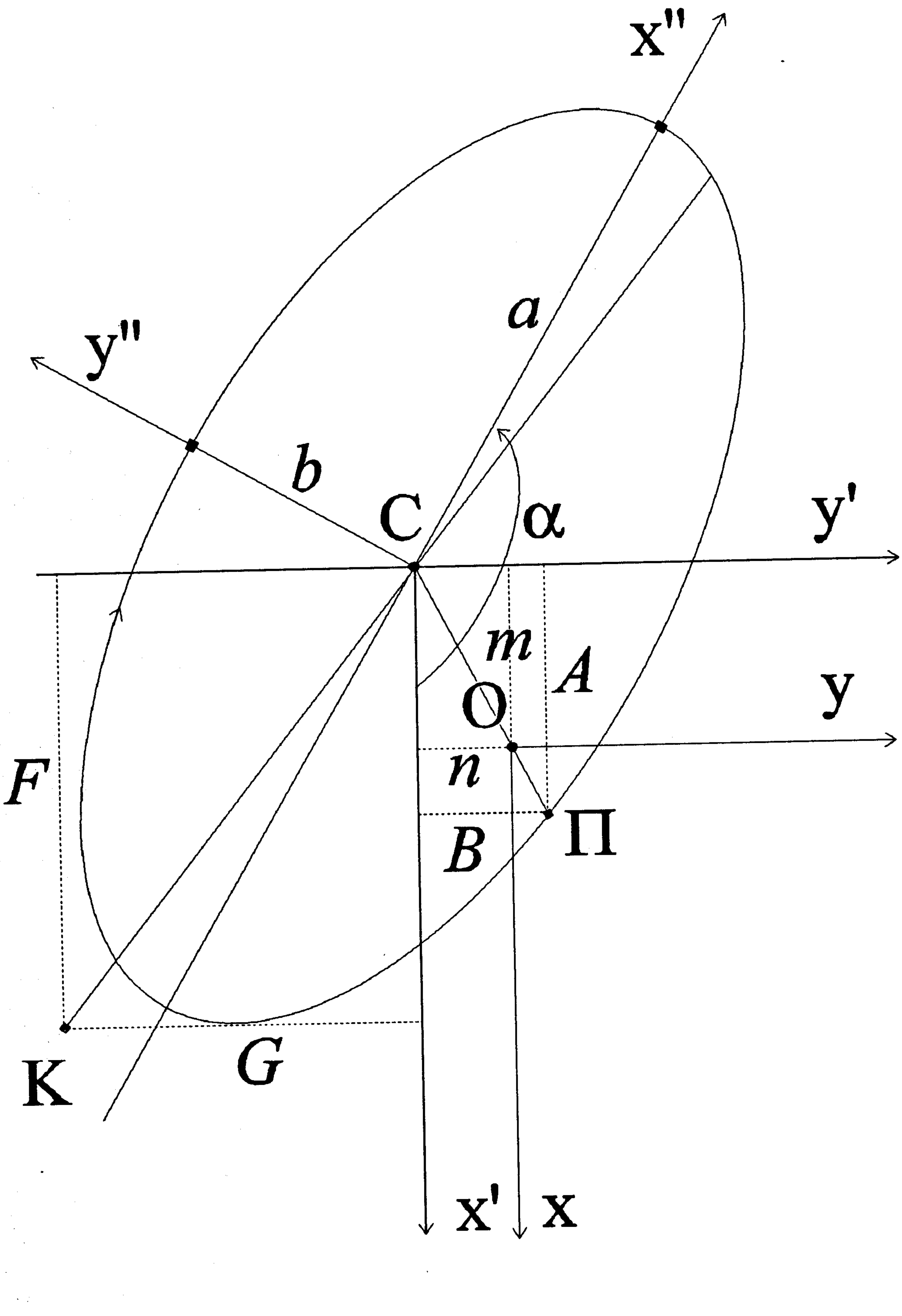

Figure 1. Geometrical meaning of the Thiele–Innes elements.

Considering that the residuals of the ephemeris can be obtained immediately after the apparent-ellipse defining, variations in the apparent-ellipse parameters are possible, which usually quickly yields a satisfactory result.

If it is concluded that from the observational material after several variations in the parameters a, b, m, n and α, one can obtain more accurate elements than by this procedure, then the elements obtained in this way can be used as initial ones in their further improvement (Eichhorn, Xu, 1990).

2. THE INNES CONSTANTS AS FUNCTIONS OF THE APPARENT–ELLIPSE AXES AND PARAMETERS OF ITS POSITION

The Innes constants A, B, F and G expressed through the apparent-ellipse axes (a, b) and the position parameters (the coordinates of the center (m, n) and rotation angle (α) with respect to the assumed coordinate system) can be simply derived taking into account their geometrical meaning (van den Kamp, 1958). In Fig. 1 the geometrical meaning of the Innes constants is given for a fictive binary. The coordinate systems in Fig. 1 Oxy and $Cx'y'$ are in the plane perpendicular to the line of sight. The main component of the system is at the point O, the axes x and x' are directed to the north, whereas the y and y' are directed to the east. The constants A and B are the rectangular coordinates of the periastron (Π) in the $Cx'y'$ coordinate system, whereas the F and G are the rectangular coordinates of the point situated on the projection of the Kepler circle, also in the $Cx'y'$ system, whose eccentric anomaly is 90°. The constants F and G are in fact a point of the conjugated radius CK enlarged by the coefficient $1/\sqrt{1-e^2}$ where $e = \mathrm{CO}/\mathrm{C\Pi}$.

The constants A and B as functions of the apparent-ellipse axes a, b and of the parameters m, v, α are then obtained from the intersection between the apparent ellipse and the straight line passing through the points O and C, i. e. from the relations

$$b^2(x'\cos\alpha + y'\sin\alpha)^2 + a^2(-x'\sin\alpha + y'\cos\alpha)^2 = a^2b^2 \qquad (2)$$
$$y'm = x'n$$

where m and n are the parameters of the translation of the coordinate system Oxy into the system $Cx'y'$ and α is the angle of rotation of the apparent ellipse (or its major semiaxis) with respect to the x (or x') axis in the east-north sense.

On the basis of relations (2) one obtains

$$A = x' = \frac{-abm}{\sqrt{u^2+v^2}} \qquad B = y' = \frac{-abn}{\sqrt{u^2+v^2}}, \qquad (3)$$

where u and v are given by the following expressions

$$u = b(m\cos\alpha + n\sin\alpha)$$
$$v = a(m\sin\alpha - n\cos\alpha). \qquad (4)$$

The true-orbit eccentricity is obtained from

$$e = \frac{\mathrm{CO}}{\mathrm{C\Pi}} = \frac{\sqrt{u^2+v^2}}{ab}. \qquad (5)$$

The geometrical meaning of the constants F and G allows their calculation in the following way: the straight line CΠ in the system $Cx''y''$ rotated by the angle α with respect to the system $Cx'y'$ has the following equation

$$y'' = \frac{-A\sin\alpha + B\cos\alpha}{A\cos\alpha + B\sin\alpha}x'', \qquad (6)$$

and its conjugated straight line

$$y'' = k_k x'', \qquad (7)$$

where

$$k_k = -\frac{b^2}{a^2}\frac{A\cos\alpha + B\sin\alpha}{-A\sin\alpha + B\cos\alpha}. \qquad (8)$$

In the system $Cx''y''$ the intersection of the conjugated straight line (7) and the ellipse is $\frac{x''^2}{a^2} + \frac{y''^2}{b^2} = 1$

$$x''_k = \pm\frac{ab}{\sqrt{b^2 + a^2k_k^2}}, \qquad y''_k = k_k x''_k. \qquad (9)$$

The sign should be chosen depending on the sense of the secondary motion: '+' for the direct, and '−' for the retrograde motion. The coordinates of the point $K(x''_k, y''_k)$ (Fig. 1) in the system $Cx'y'$ are

$$x'_k = x''_k\cos\alpha - y''_k\sin\alpha,$$
$$y'_k = x''_k\sin\alpha + y''_k\cos\alpha. \qquad (10)$$

and the constants F and G are then obtained by multiplying these constants with the coefficient $1/\sqrt{1-e^2}$, namely

$$F = \frac{x'_k}{\sqrt{1-e^2}} \qquad G = \frac{y'_k}{\sqrt{1-e^2}}. \qquad (11)$$

Further procedure of obtaining the Cambel elements from the Innes ones follows the well-known relations (Heintz, 1972).

$$
\begin{aligned}
a &= \sqrt{j+k} \\
i &= arctg\,\frac{\sqrt{a^4+m^2}}{m} \\
\omega &= \frac{z+r}{2} \\
\Omega &= \frac{z-r}{2},
\end{aligned} \tag{12}
$$

where

$$
\begin{aligned}
k &= A^2+B^2+F^2+G^2 \\
m &= AG-BF \\
j &= \sqrt{k^2-m^2} \\
z &= arctg\,\frac{B-F}{A+G} \\
r &= arctg\,\frac{B+F}{G-A}.
\end{aligned} \tag{13}
$$

The two dynamical orbital elements – period (P) and periastron-passage time (T) – can be successfully obtained by applying the least-square method from the equation of condition system

$$M_i = \frac{2\pi}{P}(t_i - T), \qquad i = 1, \ldots N, \tag{14}$$

where N is the number of observational data on which the ellipse arc is based, M_i are the mean anomalies and t_i are the times of observations.

The procedure realization is reduced to the following. The observational data (θ, ρ) after being reduced to the same epoch, are transformed into the rectangular coordinates $x = \rho\cos\theta$, $y = \rho\sin\theta$ and presented on the monitor. The apparent ellipse is drawn by choosing its parameters (a, b, m, n, α) and one tries to obtain a fit to the observational arc as good as possible. Since any change of the apparent-ellipse parameters yields an $(O-C)$ immediately, it is possible to reach satisfactory orbital elements very quickly.

If the estimated that the parameter variations do not yield essentially improved orbital elements, which is seen from the sum of the residual squares $(O-C)$ and also from the mean arithmetical error in the residuals of θ and ρ, then a certain improvement may be reached through the minimisation procedure (Eichhorn, Xu, 1990). Several examples show that "the procedure of fitting the apparent ellipse to the observational arc" proposed here is sufficiently successful so that any improvement based on the procedure of Eichhorn and Xu becomes impossible. Namely, their procedure confirms that the obtained elements correspond to a minimum sum of the residual squares $(O-C)$.

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НОВИ ПРИСТУП ИЗРАЧУНАВАЊУ THIELE–INNES-ОВИХ КОНСТАНТИ

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Ослањајући се на геометријско значење Thiele–Innes-ових константи A, B, F и G саопштавају се аналитички изрази за њих у функцији оса и параметара положаја привидне елипсе двојног система. Овај поступак успешно је примењен на више двојних система. Овако добијени елементи се могу користити као иницијални при примени Eichorn-овог поступка (Eichhorn, Xu, 1990).