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THE ROLE OF GEOMAGNETIC FIELD CONFIGURATION ON THE GPS SIGNAL PATHS THROUGH THE MAGNETOSPHERE AND IONOSPHERE

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SUMMARY: For several positions on GPS satellite orbit in respect to the geomagnetic field lines, the propagation of signals transmitted towards the same receiver at the Earth surface, is examined. The changes in electron density height distribution are estimated taking into account the plasmapause for appropriate propagation paths. The total electron content (TEC) is calculated in the ionosphere up to 650 km. The group path delay is calculated for the quasi-longitudinal and quasi-transversal propagation paths.

1. INTRODUCTION

The orbits of Global Positioning System satellites are situated at magnetospheric altitude, e.g. about 20000 km above Earth surface. Therefore, the group paths of signals (L1 = 1575.42 MHz and L2 = 1227.60 MHz), emitted towards the Earth, depend on the total electron content in the magnetosphere as well as in the ionosphere. The group path determines the delay of signal - τ, the parameter that is necessary for positioning purpose. The delay can be determined knowing the total rotation angle of the polarisation plane α, from relation $\tau = \alpha/2\pi$. Namely, a plane polarized wave propagating in the anisotropic ionized medium, magnetosphere (plasmasphere) and ionosphere, may be regarded as the vector sum of the ordinary and extraordinary magnetoionic components. Since two components travel at the different phase velocities, the plane of polarization rotates continuously along the signal's path, that is known as Faraday rotation. The total angle of rotation as given in paper of Garriott (1960), is:

$$\alpha = (C/f^2) \int_0^{h_s} N_e \, H \cos\Psi \, ds = (C/f^2) \int_0^{h_s} N_e \, H \cos\Psi \sec\chi \, dh \quad (1)$$

α – total Faraday rotation angle
$C = e^3\mu_0/(8\pi^2 \, m^2 \, c \, e_0)$,
f – frequency
N_e – electron density
H – intensity of geomagnetic field vector H
Ψ – angle between H and the direction of the wave normal
χ – zenith angle
h – height, h_s – height of the satellite

For the propagation of the signals from satellites' altitudes up to 2000 km, it is usual to use the following approximation of equation (1)

$$\alpha = (C/f^2) < H \cos\Psi \sec\chi > \int_0^{h_s} N_e(h)\, dh , \quad (2)$$

where $\int_0^{h_s} N_e(h)\, dh$ is the Total Electron Content.

TEC is defined as number of electrons in 1 m^2 crosssection along the vertical line of sight. One TEC unit is equal to 10^{16} electrons / m^2.

This approximation is not valid in the cases treated in this paper, because the geomagnetic field changes considerably along the propagation path, and from path to path examined.

The GPS signals with standard frequencies L1 and L2 propagate with delays $\tau_{L1} = \alpha_{L1}/2\pi$ and $\tau_{L2} \underset{\cdot}{=} \alpha_{L2}/2\pi = n\tau_{L1}$, respectively. The signals propagate in the same TEC conditions and in same geomagnetic field configuration. Upon these assumptions and considering the relationship (1), it can be written:

$$\begin{aligned} & e^2/(4\pi\varepsilon_0 mc)\,(1/L2 - 1/L1)TEC = \\ & = \omega_{L2}\tau_{L2} - \omega_{L1}\tau_{L1} , \end{aligned} \quad (3)$$

where ω is the angular frequency. Instead of the propagation delay determination for a single signal, the difference between delays for L2 and L1 can be used in positioning calculations. As can be seen from equation above, the value of TEC is of the prime importance in these calculations.

2. THE ELECTRON DENSITY HEIGHT PROFILE

2.1 *The diffuse equilibrium model for the Magnetosphere*

The plasmasphere is inner region of the magnetosphere, and extends above regular ionospheric layers of the Earth. The outer boundary of the region is called "plasmapause" at which the number density of charged particles drops sharply, often by an order of magnitude or more within a distance of much less than one earth radius (R_E) as measured in the equatorial plane. As might be expected, the boundary is aligned of the relatively high mobility of the particles in the magnetic field direction, as opposed to the cross field direction. There is general agreement that plasmapause position decreases as magnetic activity increases. Rycroft and Thomas (1970) showed that the plasmapause position L_{pp} was related to the geomagnetic planetary 3-hour-range index K_p at the time of measurements by

$$L_{pp} = 5.64 - (0.78 \pm 0.12)\sqrt{K_p} . \quad (4)$$

The K_p can be in the range from 0 to 9, indicating the level of the geomagnetic activity.

The electron and ion densities in the magnetosphere are represented by a field-aligned isothermal diffusive equilibrium model (Angerami and Thomas, 1964). The electron density is given by

$$N = N_0[\sum_i \xi_{i1} \exp(-z/h_i)]1/2 \quad (5)$$

$$z = r_1 - r_1^2/r - \Omega^2/2g_1\,(r^2\cos^2\Phi - r_1^2\cos^2\Phi_1) \quad (6)$$

$$h_i = \kappa T/m_i g_1 \quad (7)$$

ξ = fractional abundance of ionic species
r = geocentric distance
Ω = angular rotation speed of the Earth
g = acceleration of gravity
Φ = dipole latitude
κ = Boltzman's constant
T = temperature
m = mass.

The subscripts i refers to the i th ionic species (O^+, He^+ and H^+) and subscripts 1 refers to the reference level at 900 km altitude. Different fractional abundance of ionic species and temperatures at referent level, define the model of the magnetosphere for the seasonal and diurnal variations.

2.2 *The International Reference Ionosphere model*

Electron density distribution in the ionosphere is given by the International Reference Ionosphere model (IRI), Rawer (1981). For the description of the upper F region IRI takes into account: a) the formation of electron-ion pairs by photoionization during the daytime, b) maintenance of plasma generated by daytime ionization, during the nighttime and c) plasma flow from the plasmasphere to the ionospheric F region, during the nighttime. Generally, it can be written

$$N_e(h) = N_{max}\,F2 \cdot \exp(-Y), \quad (8)$$

where N_{max} F2 is the maximum electron density of the F2 layer and Y is the parameter which takes into account the geomagnetic latitude, solar activity and peak plasma frequency.

As suggested by Rycroft and Jones (1987), the IRI values of electron density in the ionosphere above 650 km, are overestimated. For all calculations in this paper, the midle latitude winter night-time profile under the high solar activity conditions is used.

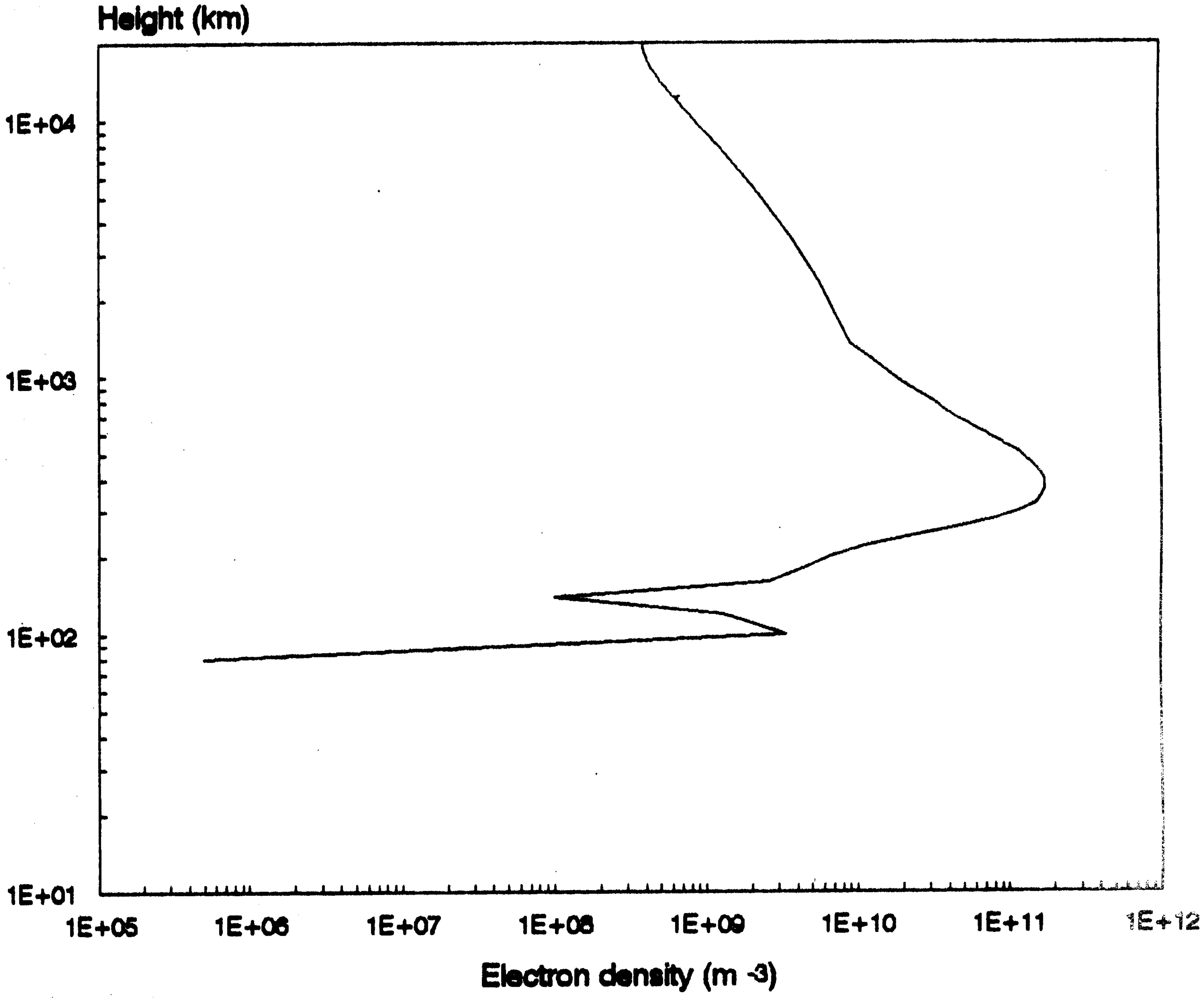

Fig. 1.

This profile is corrected to match the plasmaspheric diffuse equilibrium profile at 900 km. The fitting of the multi-region profile was obtained by Bookers method (1977). The superposition of ionospheric and magnetospheric profiles is given in Fig. 1.

3. THE RESULTS

3.1 *Determination of electron density profiles in the magnetosphere*

The height distribution of the electron density in the magnetosphere is calculated using the CDC-TRACE program, for simulation of the signal propagation in the winter-night model of the magnetosphere. The different positions of satellites in the moment of transmission are considered. The propagation direction changes from nearly quasi-longitudinal to the quasi-transversal regarding to the vector of the geomagnetic field at the position of transmitter. The receiver is supposed to be at the geomagnetic coordinates: $\Phi = 43.28°N$, $\Lambda = 102.3°E$. The five positions of the transmitter are given in Table 1, which contains geomagnetic latitude, longitude and the angle between the wave normal and vertical.

Table 1

Position	Latitude	Longitude	Inclination
1	40°N	102.3°E	181.012
2	20°N	102.3°E	187.000
3	0°	102.3°E	191.794
4	20°S	102.3°E	193.610
5	23°S	102.3°E	193.790

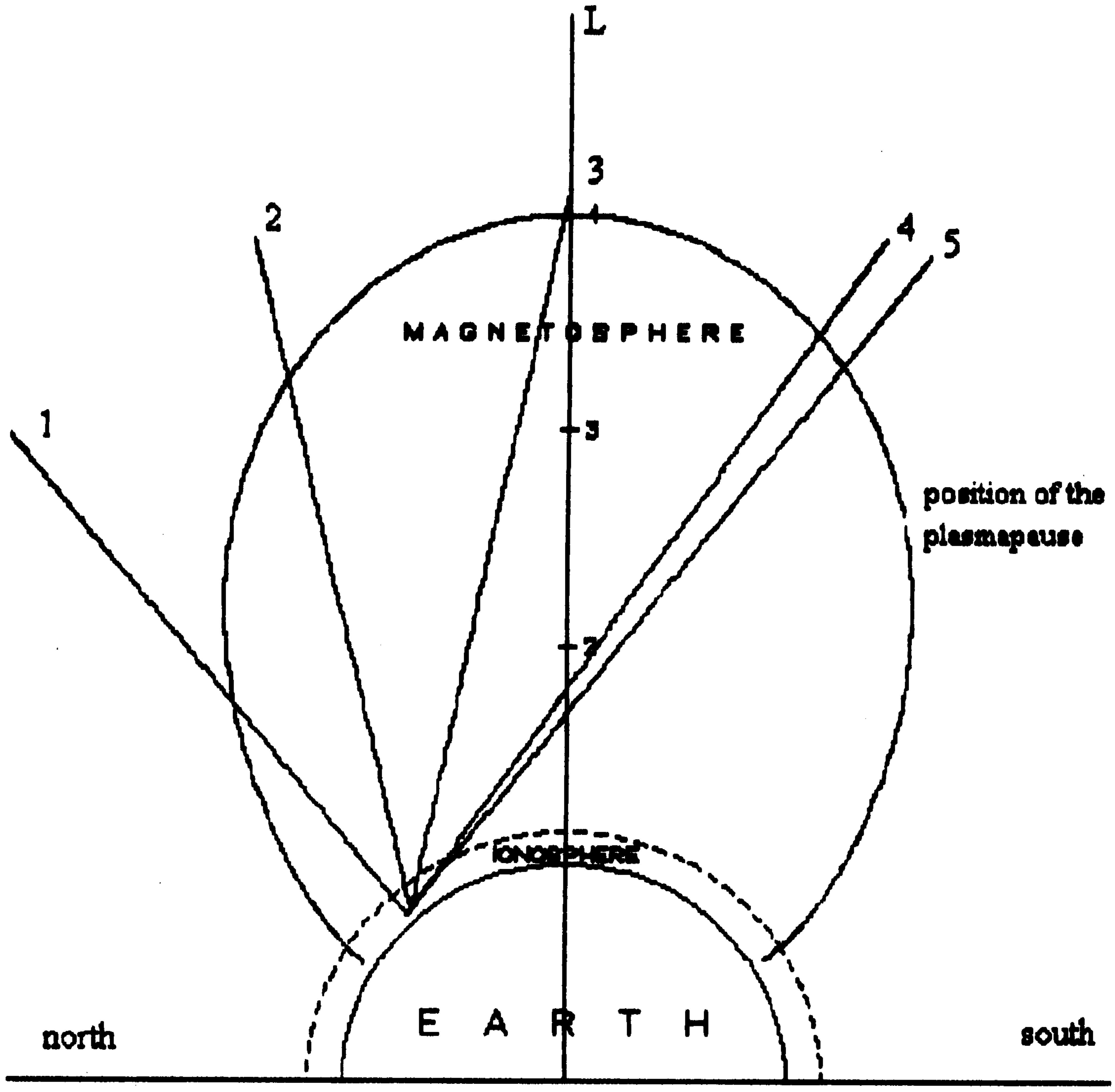

Fig. 2.

The altitude of satellite is 20000 km, above the Earth. Five positions of satellite are given in Fig.2. The $N_e(h)$ profiles in the magnetosphere are calculated for two cases: including the plasmapause position at $L = 4$ in the equatorial plane, that correlates to the regular geomagnetic condition. L is McIlwain's parameter equal to $1/\cos^2\Phi$. In the other case the plasmapause position is moved at higher L values, corresponding to the quiet geomagnetic condition. In Figs 3a and 3b, the profiles are given as calculated along the propagation paths of signals, transmitted from the satellite positions 1- 5. The influence of plasmapause on electron density profile is evident, comparing the Fig. 3a and 3b, specially for the satellite position 1.

3.2 *The calculation of total Faraday rotation angle*

The calculation of total Faraday rotation angle according to equation (1), is carried out for propagation paths corresponding to the positions 1 - 5. The horizontal isotropy regarding the electron density is assumed: $N_e(s) = N_e(h)$. Therefore, the height profiles for the magnetosphere as shown in Figs. 3a, 3b and ionospheric profile as shown in Fig.1, are considered in calculations. A centered dipole model is adopted as the geomagnetic field in CDCTRACE program. The other quantities: H, $\cos\Psi$ and $\sec\chi$ are calculated along the propagation path, by means of this program, too. It is possible to split the integral from (1) in two parts as follows:

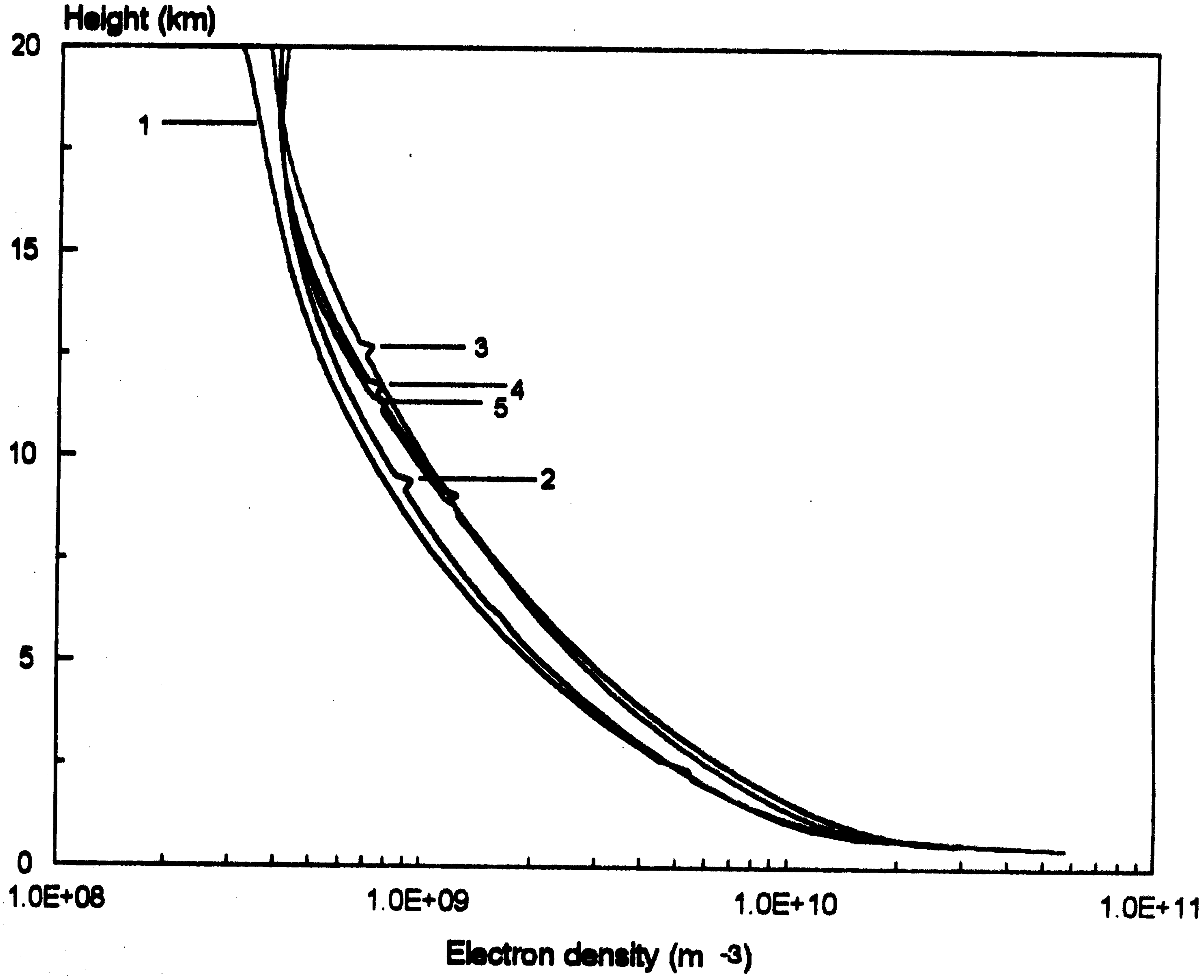

Fig. 3a.

$$\alpha = (C/f^2)\,[\int_0^{h_t} N_e\, H \cos\Psi\, dh + \int_{h_t}^{h_s} N_e\, H \cos\Psi \sec\chi\, dh] = (C/f^2)\,[< H \cos\Psi \sec\chi > \mathrm{TEC} + \int_{h_t}^{h_s} N_e\, H \cos\Psi \sec\chi\, dh] \qquad (9)$$

where the first term is an ionospheric contribution to α and the second term is the magnetospheric contribution to α. For the first term the approximation (2) is considered. In TEC calculations it is irrelevant whether the integration is carried out along the vertical drawn from the receiver, or from the subionospheric point of the transmitter. Usually, the integration is performed along the vertical through the *ionospheric intersection*, i.e. the altitude where the ray path intersects the centroid of the daytime electron density (Brown et al., 1991). The upper integration limit h_t is 650 km. The estimated TEC value is 4.77 TEC units.

In Table 2. the values of total rotation angle α, calculated from (3), are given in degrees for all propagation paths examined and for two frequencies: L1 = 1575.42 MHz and L2 = 1227.6 MHz.

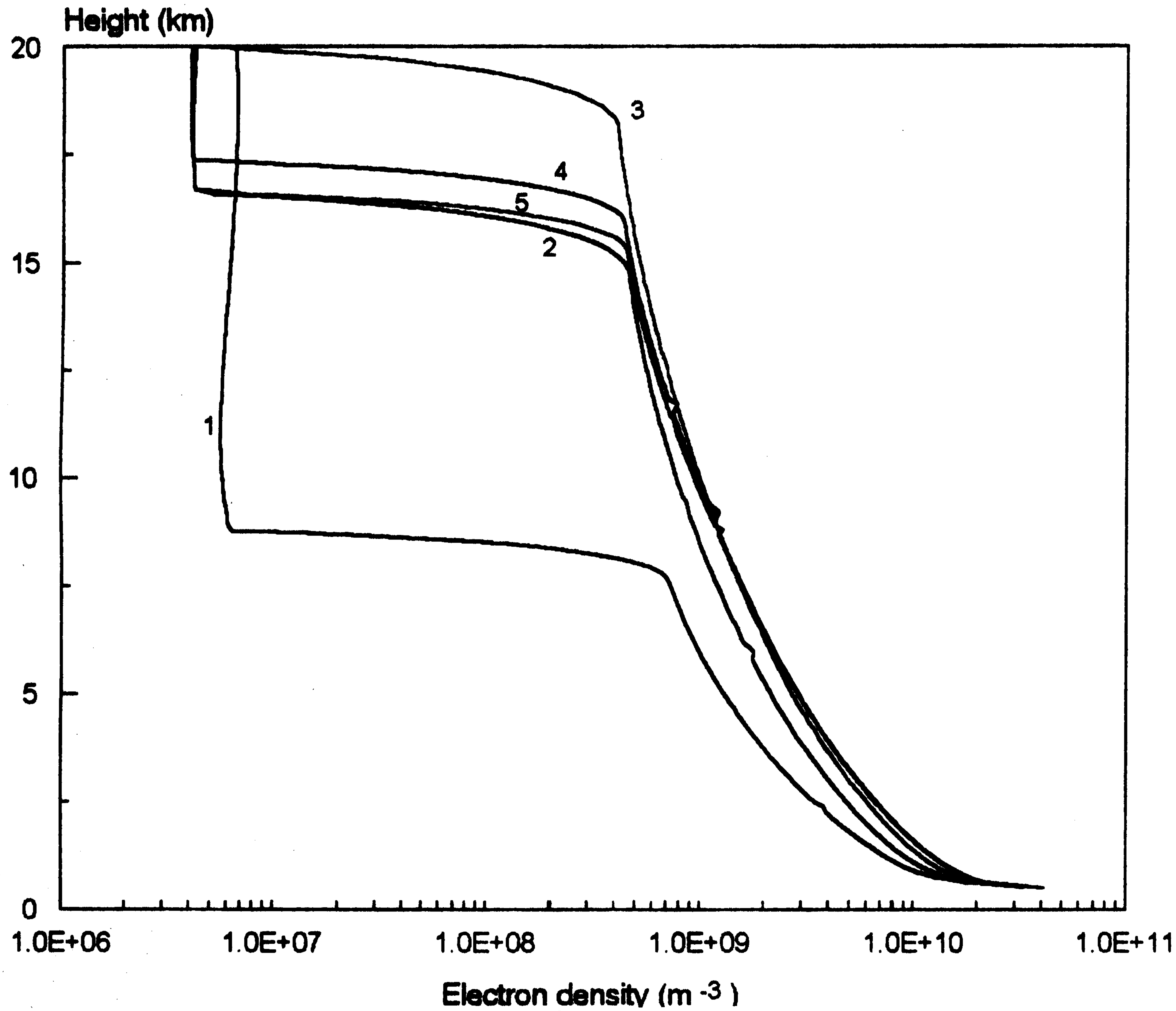

Fig. 3b.

Table 2

Satellite position	Plasmapause included		Plasmapause not included	
	L1	L2	L1	L2
40°N	1.298	2.138	1.212	1.995
20°N	1.383	2.279	1.364	2.246
0°	1.284	2.114	1.277	2.103
20°S	1.142	1.881	1.028	1.693
23°S	0.997	1.642	1.007	1.659

3.3 *The calculation of group paths*

The group paths for the propagation through magnetosphere are calculated by means of CDCTRACE program and the group paths for the propagation through the ionosphere by the Jones-Stephenson 3D ray tracing program. The incidence angle of the ray coming from the magnetosphere, plus 90°, is the angle of "elevation" in the further propagation in the ionosphere. The propagation conditions are connected through superposed electron density profile, as mentioned above. The determined group paths are given in km and presented in Table 3.

Table 3

Satellite position	Plasmapause included		Plasmapause not included	
	L1	L2	L1	L2
40°N	20026.11	20027.74	20077.65	19975.42
20°N	20690.17	20682.66	20685.31	20687.82
0°	22171.80	22191.59	22188.62	22187.90
20°S	24183.07	24361.71	24203.09	24203.54
23°S	24583.58	24660.51	24589.26	24589.94

CONCLUSION

The good knowledge of electron density distribution up to the magnetospheric heights is useful in the predictions of the conditions determining the positioning by GPS. Usually, the predictions are obtained by calculation of TEC up to the 2000 km. It is the aim of this paper to show that the magnetospheric part of the signal's path affects the propagation conditions and has to be taken into account for the predictions.

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УЛОГА КОНФИГУРАЦИЈЕ ГЕОМАГНЕТСКОГ ПОЉА НА ПУТАЊЕ GPS СИГНАЛА КРОЗ МАГНЕТОСФЕРУ И ЈОНОСФЕРУ

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Оригинални научни рад

Кратак садржај: Приказани су резултати истраживања простирања сигнала са пет различитих положаја GPS сателита у односу на линије геомагнетског поља, ка истом пријемнику на површини Земље. Промене у висинској расподели електронске густине рачунате су за постојање плазмапаузе на L = 4 и за случај када је она померена на веће вредности L параметра. Израчунат је укупни електронски садржај до 650 km у јоносфери. Групно кашњење је рачунато за квазилонгитудиналне и квазитрансферзалне путање простирања.