Bull. Astron. Belgrade № 151 (1995), 1 – 5

UDC 524.47–54
Original scientific paper

ON THE VALUES OF AGEKYAN'S FACTOR IN STELLAR SYSTEMS

S. Ninković

Astronomical Observatory, Volgina 7, 11050 Belgrade, Yugoslavia

(Received: January 9, 1995)

SUMMARY: The behaviour of Agekyan's factor - the quantity $\frac{<m^{3/2}>}{<m>^{3/2}}$ appearing in the formula of Agekyan's criterion for estimating the importance of irregular, resp. regular forces, in stellar systems - is studied. It is found that in the case of a mass distribution of stars expected on the basis of stellar statistics the value of this quantity tends to be about 2. In general, the value of Agekyan's factor exceeds one, increasing gradually with N (number of stars in a stellar system) towards 2 expected for N high enough. According to Agekyan's criterion with taking into account the present results a classification of gravitationally bound stellar systems consisting of stars based on stellar dynamics is proposed.

1. INTRODUCTION

Agekyan's criterion (Agekyan 1962) was formulated for the purpose of estimating the role of irregular, resp. regular forces in stellar systems. It is characterised by the presence of a factor depending on the mass distribution in a stellar system (the present author proposes the name of Agekyan's factor for it). Therefore, in order to estimate the role of these forces in a stellar system with given N (total number of stars) one should estimate the value of Agekyan's factor. This is important because then it becomes possible to compare the results of application of Agekyan's criterion to the case of other criterions developed for the same purpose. In this way we have approached the final goal, the formulation of a dynamical classification of bound stellar systems according to their N.

Bearing this in mind the task of the present paper will be to study the values of Agekyan's factor in systems containing stars and to try to formulate a classification of bound stellar systems based on Agekyan's criterion.

2. PROCEDURE

The role of irregular, resp. regular forces, in stellar systems in Agekyan's criterion is estimated through the ratio of the volume of a stellar system where the former dominate over the latter ones to the entire volume of the system. The corresponding formula (Agekyan, 1962 - form. (5)) is

$$2N^{-1/2}A, \tag{1}$$

$$A = \frac{< m^{3/2} >}{< m >^{3/2}}.$$

The designation m refers to the mass of an individual star and $< >$ means that the mean value is taken over the entire system. It is seen that the formula is a product of two dimensionless factors where the first one is a monotonous function of N and the second one denoted A is what has been called Agekyan's factor above.

The dependence of the role of the two types of forces on N is clearly indicated through the first factor, whereas the presence of the second one makes this dependence less clear. The cases of some systems of galaxies where members exist a few orders of magnitude as massive as the other, are well known; also there are possibilities of existence of very massive clouds of interstellar matter. In all these situations Agekyan's factor can attain extremely high values. However, these cases, being really rather extreme, will not be treated in the present paper which is confined to ordinary gravitationally bound systems consisting of stars only. This circumstance enables one to establish a relationship between the values of Agekyan's factor and N.

3. RESULTS

In a system containing a sufficiently large number of stars the value of Agekyan's factor can be obtained through the law of mass distribution. As well known, the most frequently used law of mass distribution is a power law where the best known case is Salpeter's one in which the number of stars within a narrow mass interval decreases with the power value of 2.35 (e. g. Marochnik, Suchkov, 1984 - p. 200). It is clear that in such a situation the mean values also depend on the mass limits assumed for the stars. Taking into account the mass values usually cited for ordinary stars one assumes here the values of 0.1 $M_\odot$ and 50 $M_\odot$ as the lower and upper limits, respectively. Using them one easily obtains 2.19 for Agekyan's factor in the case of Salpeter's law. Other variants of the power law are also treated and it is interesting to note that the highest value for Agekyan's factor (about 2.8) appears when the exponent is equal to -1.8. This is the consequence of the mean value of about 1 $M_\odot$ (to be compared to 0.34 $M_\odot$ in the case of Salpeter's law).

On the other hand in the book by Marochnik and Suchkov, in the same chapter, the authors mention some reserves concerning the application of Salpeter's law, in the sense that the fraction of low massive stars (e. g. less massive than 2 $M_\odot$) yielded by it is too small, whereas the one of high-mass stars (say, exceeding 10 $M_\odot$) appears to be too large. Therefore, it becomes necessary Salpeter's law to be amended. On the basis of what is given in the book by Marochnik and Suchkov (form. 13.1.9, p. 202) one finally finds that the value of Agekyan's factor suggested by stellar statistics should be about 2. As an example it can be also stated that a combination of a power law of -1.75 within the interval $0.1 M_\odot - 1 M_\odot$ and Salpeter's law beyond this interval yields $A = 2.01$ with the mean mass of 0.588 $M_\odot$.

On the other hand, Agekyan's factor cannot be less than 1, this being the case, as easy to see, when all the stars in a stellar system have the same mass. It is clear that the larger N is, the lower is the probability of all star masses to be equal. Strictly speaking it is impossible to find two stars with exactly equal masses, however bearing in mind the constraints of mass determination one may consider two, or more stars, with masses nearly equal (equality within error limits) as if they were equal exactly. Therefore, one may conclude that normally Agekyan's factor lies within the interval $1 < A \leq 2$.

It is, certainly, of interest to study the relationship between Agekyan's factor and N. Though, at first glance, it may seem that in systems containing a sufficiently large number of stars, A is expected to be about 2 (based on what has been established above) with a small dispersion and at the same time that in systems containing a smaller number of stars one can only expect A to be greater than 1, some kind of relationship based on probability theory can be found.

It seems according to modern stellar statistics that the frequency of systems of stars is inversely proportional to to the number of stars per system so that the double stars are the most frequent (e. g. Anosova, 1990). This circumstance enables to imagine a sample of many double stars and then to undertake the calculation of the mean value of Agekyan's factor in it. The rate of stars of different mass will, certainly, be reflected so that the combinations containing rare stars will also be rare, or less frequent. In this way it will be possible to estimate the expected value of Agekyan's factor for double stars.

For this purpose the individual stars are divided into several mass groups where the limits of these groups are chosen following the formula presented by Marsakov and Suchkov (1984 - (13.1.9), p. 202). The frequency of each group is determined according to the mass-distribution law for individual stars assumed above. The presentation of the groups is given in Table 1.

Table 1. Frequency of the chosen star groups

Group	Mass limits ($M_\odot$)	Frequency
I	0.1 – 0.4	85%
II	0.4 – 1.0	11%
III	1 – 2	2.7%
IV	2 – 10	1.3%
V	10 – 50	0.02%

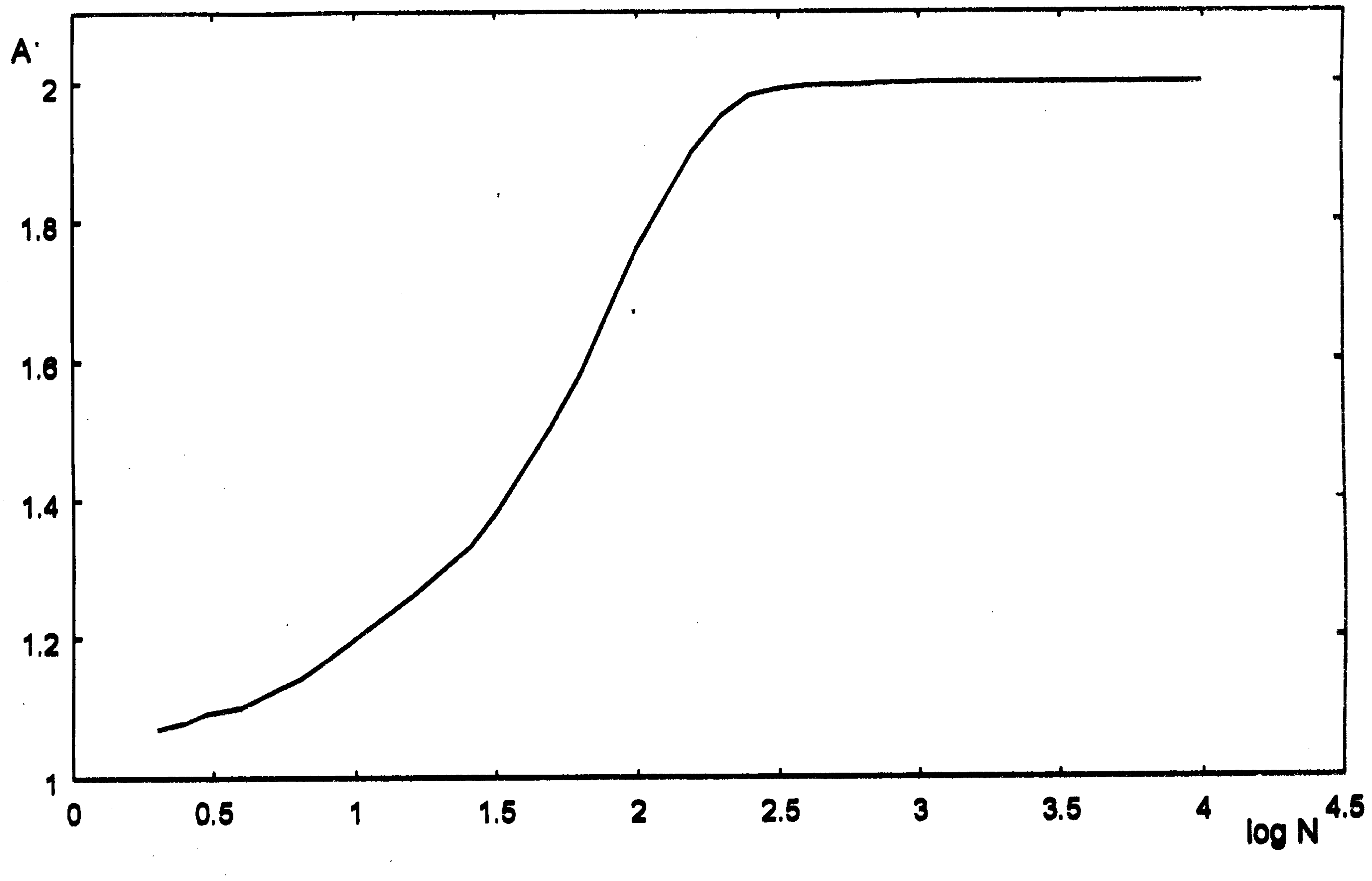

Fig. 1. Agekyan's factor versus number of stars.

For the case of double stars Agekyan's factor is determined for all possible combinations of the groups. As the first step its mean value is estimated within each combination (smoothing procedure), then after assigning to the combination its weight which is proportional to the probability where the latter one directly depends on the frequency (Table 1), one obtains the expected value of Agekyan's factor for double stars as a weighted mean. The result is 1.07. By applying analogous procedures the expected values of Agekyan's factor are estimated also for the systems of higher multiplicity. As expected these values increase with N increasing. The corresponding dependence is presented in Fig. 1. As seen from the figure at N about 250 Agekyan's factor already reaches its statistical value of 2.

The substitution in (1) of the results presented in Fig. 1 enables one to estimate the rate of the irregular forces in systems of stars on the basis of Agekyan's criterion. Some values of this rate can be indicated as critical. For example, the rate of the irregular forces (1) reaches the value of 1 at N between 5 and 6. If the role of the irregular forces is considered as negligible when (1) yields values less than, say, 5 %, then this condition will be satisfied in systems with N greater than 6400.

It is also possible to propose a classification of gravitationally bound systems of stars which is based on Agekyan's criterion through the values found here for Agekyan's factor:

multiple stars - $2 \leq N \leq 5$ - rate of irregular forces exceeds 1;

systems intermediate between multiple stars and open clusters - $5 < N \leq 100$ - rate of irregular forces less than 1, but Agekyan's factor less than 2;

open clusters - $100 < N < 10000$ - rate of irregular forces not negligible, Agekyan's factor about 2;

globular clusters and galaxies - $N \geq 10000$ - rate of irregular forces negligible.

The limits in N, such as the upper one for the intermediate systems and further on do not correspond exactly to the values given in Fig. 1; they are rather suitable round numbers sufficiently close to the critical ones. The names of the systems appearing in the proposed classification are connected with our intuitive concepts of the given systems of stars. In our intuition based on stellar dynamics multiple stars are expected to be the systems where the irregular forces prevail. On the other hand open clusters are interpreted as systems where statistical effects are expected to be of importance, therefore Agekyan's factor in them should be near its statistical value, found here to be about 2. As a subclassification one may add the notions of small open clusters ($N \sim 10^2$) and large open clusters ($N \sim 10^3$) where in the latter ones the rate of the irregular forces is less than 10 %. Finally, globular clusters and galaxies are understood as systems in which the irregular forces are generally negligible.

4. DISCUSSION AND CONCLUSIONS

It should, certainly, be pointed out that the classification proposed here cannot be understood as universal since it is based on one criterion only - that of Agekyan. The problem of classifying dynamically the gravitationally bound systems of stars is important and it deserves further thorough studies taking into account other results.

In any case what results from Agekyan's criterion should be compared to the results following from another criterion. As a suitable criterion for estimating the importance of the irregular (regular) forces in systems of stars one can mention the amount of the relaxation time of a system expressed in terms of its crossing time (e. g. Binney, Tremaine, 1987 - form. (8.1), p. 489). In this formula the ratio relaxation time to crossing one depends on N only. Its reciprocal value may be compared to results of applying (1) after taking into account the relationship between A and N established here. The agreement is satisfactory since the values yielded by the two formulae are of the same order of magnitude. This is enough with regard to the approximative character of both formulae. The best agreement is reached in the domain of $N \sim 10^2$ where the values yielded by the two formulae are approximately equal, however for $N \sim 10^1$ the formula given by Binney and Tremaine yields systematically higher amounts, whereas in the domain of $N \sim 10^3$ the situation is converse.

On the basis of the present estimates it is concluded that in the gravitationally bound systems consisting of ordinary stars (covering mass interval 0.1 $M_\odot$ - 50 $M_\odot$) the values of Agekyan's factor are expected to exceed 1 and to increase with increasing N (total number of stars) to attain the value of about 2 at N sufficiently high (say ≥ 250). This conclusion has a rather statistical character.

The amounts of Agekyan's factor, obtained in such a way and substituted in (1), enable a more direct application of Agekyan's criterion to be made for the purpose of estimating the role of the irregular, i. e. regular, forces in such systems. The agreement between the estimates resulting from (1) and those based on the ratio of the relaxation time to the corresponding crossing one are satisfactory. Finally, according to the results obtained here by using (1) a dynamically based classification of gravitationally bound systems consisting of stars is proposed.

Acknowledgments - This work has been supported by Ministry for Science and Technology of Serbia through the project "Physics and Motions of Celestial Bodies".

The author is grateful to Mr R. Pavlović for his unselfish aid in the technical preparation of the manuscript.

REFERENCES

Agekyan, T. A.: 1962, in *Kurs astrofiziki i zvezdnoj astronomii*, ed. A. A. Mikhajlov, gos. izdat. fiz.-mat. lit., Moskva, vol II, ch. XX, p. 528.

Anosova, J. P.: 1990, *Bull. Obs. astr. Belgrade*, **142**, 1.

Binney, J., Tremaine, S.: 1987, *Galactic Dynamics*, Princeton University Press, Princeton, New Jersey.

Marochnik, L. S., Suchkov, A. A.: 1984, *Galaktika*, "Nauka", glav. red. fiz.-mat. lit., Moskva.

О ВРЕДНОСТИМА АГЕКЈАНОВОГ ЧИНИОЦА У ЗВЕЗДАНИМ СИСТЕМИМА

С. Нинковић

Астрономска опсерваторија, Волгина 7, 11050 Београд, Југославија

УДК 524.47–54
Оригинални научни рад

Проучава се понашање Агекјановог чиниоца, величине $\frac{<m^{3/2}>}{<m>^{3/2}}$ која фигурише у формули Агекјановог критеријума за процену значаја ирегуларних, одн. регуларних сила, у звезданим системима. Нађено је да у случају расподеле звезда по масама која се очекује на основу звездане статистике вредност ове величине има тенденцију да буде око 2. Уопште, вредност Агекјановог чиниоца је већа од један и постепено се повећава са повећањем N (број звезда у звезданом систему) према 2 за довољно велико N. У складу са Агекјановим критеријумом, узимајући у обзир овде добијене резултате, предлаже се једна класификација гравитацијски везаних система који се састоје од звезда заснована на звезданој динамици.