## RAY-TRACING STUDY ABOUT INITIAL CONDITIONS OF MULTICOMPONENT WHISTLERS IN THE ASYMMETRIC MODEL OF THE MAGNETOSPHERE

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SUMMARY: Ray-tracing calculations about initial and final latitude ranges for trapping 3 kHz ray in the ducts are presented. In the summer-winter model of the magnetosphere from L>2.9 to L<4.6 eight ducts are incorporated. For started rays in the summer-dawn hemisphere the initial latitude ranges have average value  $0.1^{\circ}$ , and are moved equatorward from duct centers. The final latitude ranges for downcoming wave in the winter-night hemisphere have average values grater than initial latitude ranges. For ducts located at L>4.4 the final latitude ranges are spread around the duct centers. Trapping of upgoing whistler-mode waves through the side of ducts is presented. Escaping of downcoming waves occurs also through the side of ducts. The minimum value of the refractive index along the duct causes trapping and escaping of VLF wave through the side of the duct.

## 1. INTRODUCTION

There is strong evidence that the paths followed by VLF whistler-mode signals propagating in the magnetosphere between conjugate ground points are discrete in nature. Ground-to-ground propagation is usually interpreted as evidence of guiding VLF waves by density irregularities or "duct" that extend without interruption over most of the length of field aligned paths (Smith, 1961; Angerami, 1970; Bernhardt, 1979). Studying whistlers observed at ground station may provide information on the conditions under which such ducts can exist and to what geocentric distances in the magnetosphere they can extend (Smith and Angerami, 1968; Bernhardt and Park, 1977; Inan and Bell, 1977; Thomas, 1978; Carpen-

ter 1981; Carpenter and Šulić, 1988; Strangeways, 1991).

The purpose of this paper is to report about enter and exit points of the components of multicomponent whistlers. The components of the whistler, propagate through different ducts, located in the summer-winter model of the magnetosphere. In this model of the magnetosphere electrons and hydrogen, helium and oxygen ions densities are presented by a magnetic field-aligned isothermal diffusive equilibrium model (Angermani and Thomas, 1964). This model is generally employed to determine the plasma composition at any point, in terms of the base level (900 km) altitude composition of field line passing through the point. A centered dipole model is adopted as the geomagnetic field in the magnetosphere.

The effect of asymmetry in magnetospheric

plasma distribution about equatorial plane must exist at all times except perhaps at equinox. This means that the equatorial electron density for winter conditions cannot be independent of the summer conditions in the conjugate hemisphere. Strangeways (1982) has developed a ray-tracing model of the inner magnetosphere (L < 6) for a plasma distribution, asymmetric about the equatorial plane, thus representing summer and winter conditions in the two hemispheres. At the reference height at 900 km, the oxygen ion concentration and electron density are taken to vary by factors of ten and two respectively between hemispheres. The concentrations of hydrogen and helium ions of the reference level are chosen to ensure electron density continuity across the equatorial plane.

#### 2. WHISTLER DATA

The base of this analysis were whistlers recorded at Siple Station (L=4.3) Antarctica, on July 04, 1982, during local dawn time sector. Whistlers were frequent at that day. Whistlers were recorded under very quiet geomagnetic conditions. Maximum Kp value in 12 hours preceding the local dawn on July 04, 1982, is 1. Characteristic features of the data, including triggering of VLF emissions bursts were identified and equatorial electron density profile was estimated, using the methods of Park (1972). The dawn sector was selected because the wave-associated burst precipitation was observed (Sulic, 1988).

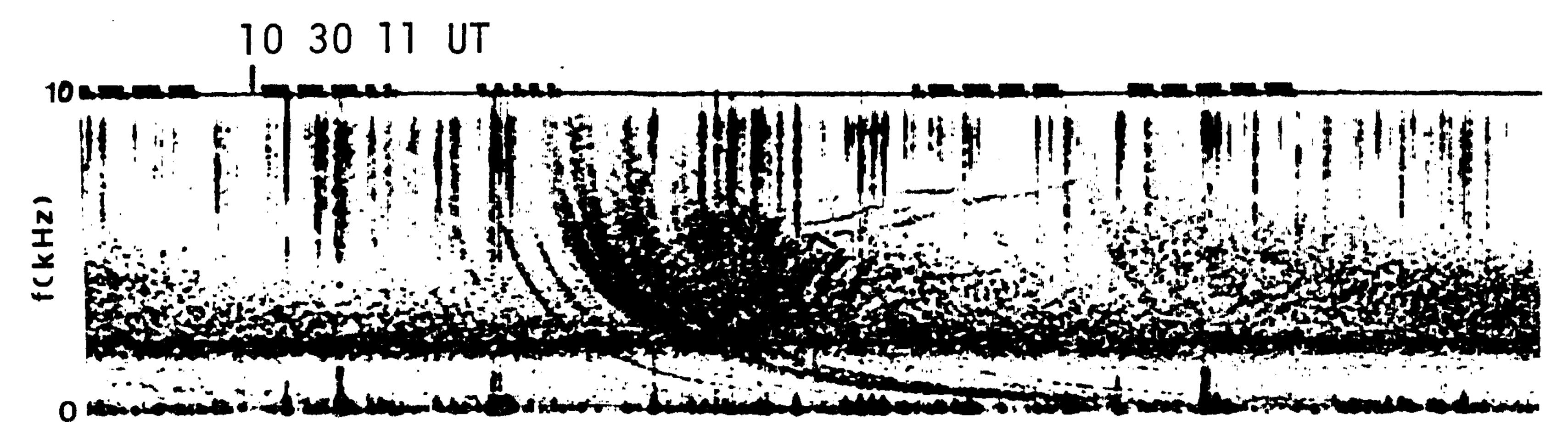


Fig. 1 Spectrogram of multicomponent whistler recorded at Siple Station, Antarctica on July 04, 1982.

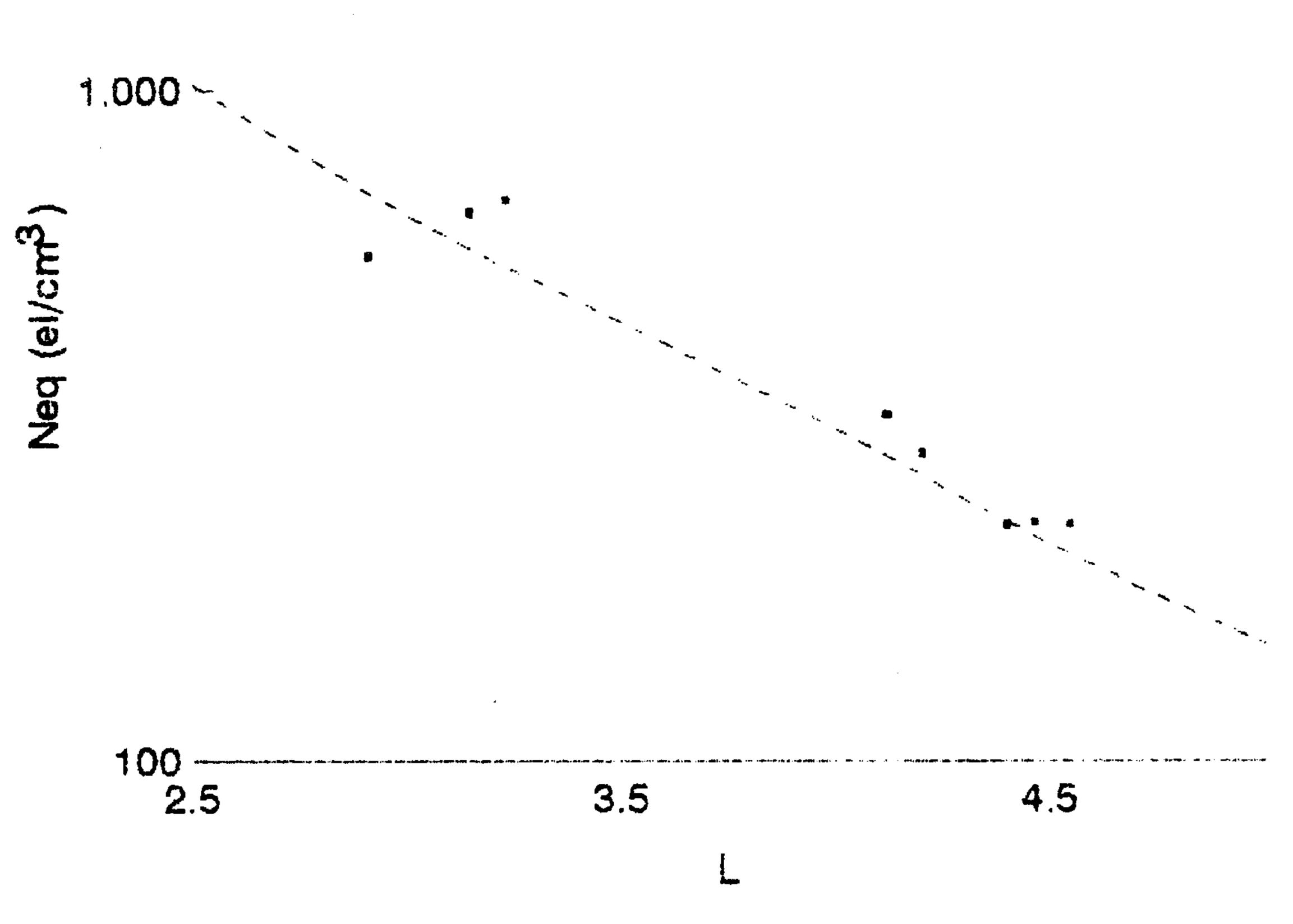


Fig. 2 Magnetospheric equatorial electron density values deduced from whistlers recorded at Siple Station during local dawn on July 04, 1982. The dashed curve is a reference profile representing median density values observed from Siple on June, 1973 (from Park et al., 1978).

Figure 1 presents spectrogram of multicomponent whistler in the frequency range 0 - 10 kHz, recorded at Siple Station, Antarctica on July 04, 1982. All components of the whistler accompanied by triggering of VLF emissions, propagated inside

plasmasphere.

Figure 2 shows the profile of the equatorial electron density, deduced from whistlers recorded during local dawn on July 04, 1982. The dashed curve is a reference curve, it represents median equatorial

density values in the plasmasphere observed from Siple Station in June 1973, for the local dawn (Park et al., 1978). The profile is consistent with Figure 1 showing all components of the whistler (data points) inside plasmasphere.

#### 3. WORKING MODEL

Our aim is to obtain an approximate enter and exit points for rays trapped simultaneously in eight ducts in asymmetric model of the magnetosphere. As explained above, we use the summer-winter night model of the magnetosphere to perform a ray-tracing calculation for rays inside plasmasphere.

The ray path parameters have been computed by using CDCTRACE program (Strangeways, 1992) adapted for PC 486. The program performs raytracing in a two-dimensional meridian plane, using a centered dipole model for the magnetic field. One of the main components of the input data for the raytracing program is the field-aligned electron density model. By adjusting the various input parameters of the program it is possible to input a field-aligned electron density enhancement for different L values and to form ducts. A duct with Gaussian cross-section could be incorporated into this asymmetric model that gives an increase of electron density inside duct. The enhancement at any position across the width of the duct was calculated from

$$\Delta = \delta \exp(-x^2/2\sigma_d^2) \tag{1}$$

where x is the separation (in km) between the field lines corresponding to the ray position and that of the center of the duct in the equatorial plane,  $\delta$  is the fractional plasma density enhancement at the duct center and  $\sigma_d$  is the distance either side of the duct where the density has fallen to  $\delta/\sqrt{e}$ . The distance for the enhancement to fall to a tenth of its maximum value, which is considered to be an indication of the effective width of the duct, is  $2.15 \sigma_d$  either side of the duct center.

In this study for eight ducts located at L = 2.906; 3.143; 3.227; 4.115; 4.199; 4.396; 4.462 and 4.542 the enhancement  $\delta$  was taken to be 0.15 and the value of  $\sigma_d$  was 50 km, corresponding to an effective width of 215 km. The ducts were assumed to extend down to 300 km in the initial summer hemisphere and were terminated at above 300 km in the conjugate winter night hemisphere. Bernhardt and Park (1977) have suggested that VLF ducts might extend down to 300 km altitude at night but usually terminate above 1800 km during day in winter and equinoxes. In summer ducts terminated above 1000 km altitude at all local time. Strangeways (1982) suggested that over northern America where constant particle precipitation associated with magnetospheric amplification of power line harmonics in the VLF range existes, it may maintain duct enhancement down to 300 km even in the summer hemisphere. The base for this study are whistlers originated over northern America. On spectrograms the triggering of VLF emissions bursts were estimated during the local time sector dawn. That is the main reason for modeling the ducts to extend down at 300 km altitude in the summer dawn hemisphere. The ducts reached the full enhancement at 2100 km altitude in both hemispheres disregarding the seasonal variation in the hemispheres.

The effect of plasma asymmetry was also investigated for a summer-winter model in that way that the ratio of oxygen ion concentration at the reference height of 900 km in one hemisphere to that in the other hemisphere was changed for each duct. The final value of this ratio for each duct was varied until the calculated electron density at reference 1400 km altitude in the summer hemisphere was similar to the data obtained by ISIS 2 satellite. The satellite data were acquired at 1400 km altitude at dawn and dusk in the sunlit northern hemisphere within 150 longitude of the Roberval Station (~76°W; L ~ 4.3), that is the conjugate station to Siple Station, during 6 days in June 1973. Magnetic conditions changed gradually from moderately disturbed to quiet during the interval of the study (Foster at al., 1977). These satellite data of electron density and oxygen ion density against L value are very useful in modeling asymmetric model of the magnetosphere.

Ray-tracing calculations were performed for 3 kHz waves that had initial vertically pointing wave normals at 300 km altitude over a range of initial dipole latitudes from 52°N to 61.5°N.

#### 4. RESULTS

The distance between the first duct at L = 2.906 and the eighth duct at L = 4.542 is 950 km at 300 km altitude. Estimation of the initial positions for 3 kHz rays to be trapped in eight ducts simultaneous in the same way is very important for studying trapping conditions for multicomponent whistlers in the asymmetric model of the magnetosphere. The results are summarized in Table 1. Characteristic features of the duct centers, as L value and dipole latitude at 300 km altitude, are given in the first and second column of Table 1, respectively. Initial latitude ranges for each duct is represented in the third column. The beginning and ending initial latitude and the difference between them are third column. The width of the initial latitude range varies from  $\Delta\Theta_i = 0.061$  to  $\Delta\Theta_i = 0.129$  for ducts L = 3.227 and L = 4.462, respectively. The 3 kHz ray is found to become trapped through the duct side when starts from 0.40 to 1.20 equatorward of duct centers for different ducts, in summer hemisphere during local dawn.

Ray-tracing calculations were done changing the latitude inside initial range by step of 0.0010 to define the final latitude range and final wave normal

angle to the vertical. The results about the final latitude ranges are given in the fourth column. The width of the final latitude range at 300 km altitude in the winter night hemisphere is larger than the initial latitude range for each duct. The width of the final latitude range for 3 kHz ray varies from  $\Delta\Theta_f$  =  $0.488 \text{ to } \Delta\Theta_f = 2.683 \text{ for duct L} = 2.906 \text{ and L} =$ 4.199, respectively. Analyzing the values of the final latitude range according to the dipole latitude of the duct center, the conclusion is that for the first four ducts final latitudes are moved equatorward from the duct center. The rays from the other four ducts can reach at 300 km altitude in the winter night hemisphere inside larger final latitude ranges, which are spread around the dipole latitudes of the duct center.

Any whistler mode wave originated by lightning source in the atmosphere enters the magnetosphere through the ionosphere, with an essentially vertical wave normal angle. After a ducted whistler wave propagates through the magnetosphere and reaches the conjugate ionosphere the reverse problem exists. If the final wave normal of the ray is inside the ionospheric transmission cone it can propagate through the ionosphere into Earth-ionosphere wavegaide (Helliwell, 1965). For 3 kHz ray average values of the transmission cone for these analyzed ducts are from  $\pm 1.15^{\circ}$  to  $\pm 1.8^{\circ}$ . In the fifth column of the Table 1 are given average values of the final wave normal angle to the vertical at 300 km altitude and average value of the transmission cone in the winter night hemisphere. For ducts located from L = 2.906to L = 4.199 it is evidently that for the most part of the initial latitude ranges the final wave normal angles to the vertical lie inside the transmission cone. The final wave normal angles to the vertical lie out- function of the initial direction of the wave normal

side the transmission cone for ducts L = 4.462 and L= 4.542. Even so small step of 0.001° in changing the input initial latitude for ray-tracing calculations has not resulted in any ducted ray with final wave normal angle to the vertical inside the transmission cone. The values of the final wave normal angles to the vertical are spread from -14° to -3°. For a groundobserved whistler to be observed from the seventh and eighth ducts, scattering by small-scale irregularities in the F-region or reflection by large-scale gradients in the lower ionosphere would be required to enable the rays to penetrate the Earth-ionosphere waveguide (Strangeways, 1982).

The sixth column contains the height of enter points of 3 kHz ray in the ducts. The trapping of VLF waves through the side of the duct is consequence of the minimum value of the refractive index in that region of the summer hemisphere. The seventh column contains value of the height of the exit points. Escaping ducted VLF waves through the duct side in the winter hemisphere occurs in the region of the minimum refractive index in the winter hemisphere. The values of height of enter and exit points increase with increasing the L value of the duct.

The direction of the wave normal is very important for trapping VLF waves in the duct. Raytracing calculations have been done for 3 kHz ray started at 300 km altitude in the summer dawn hemisphere to define initial latitude range with changing the direction of the wave normal. The initial wave normal direction has been moved from geomagnetic field direction to vertical direction. Figure 3 shows the distribution of the initial latitude range as the

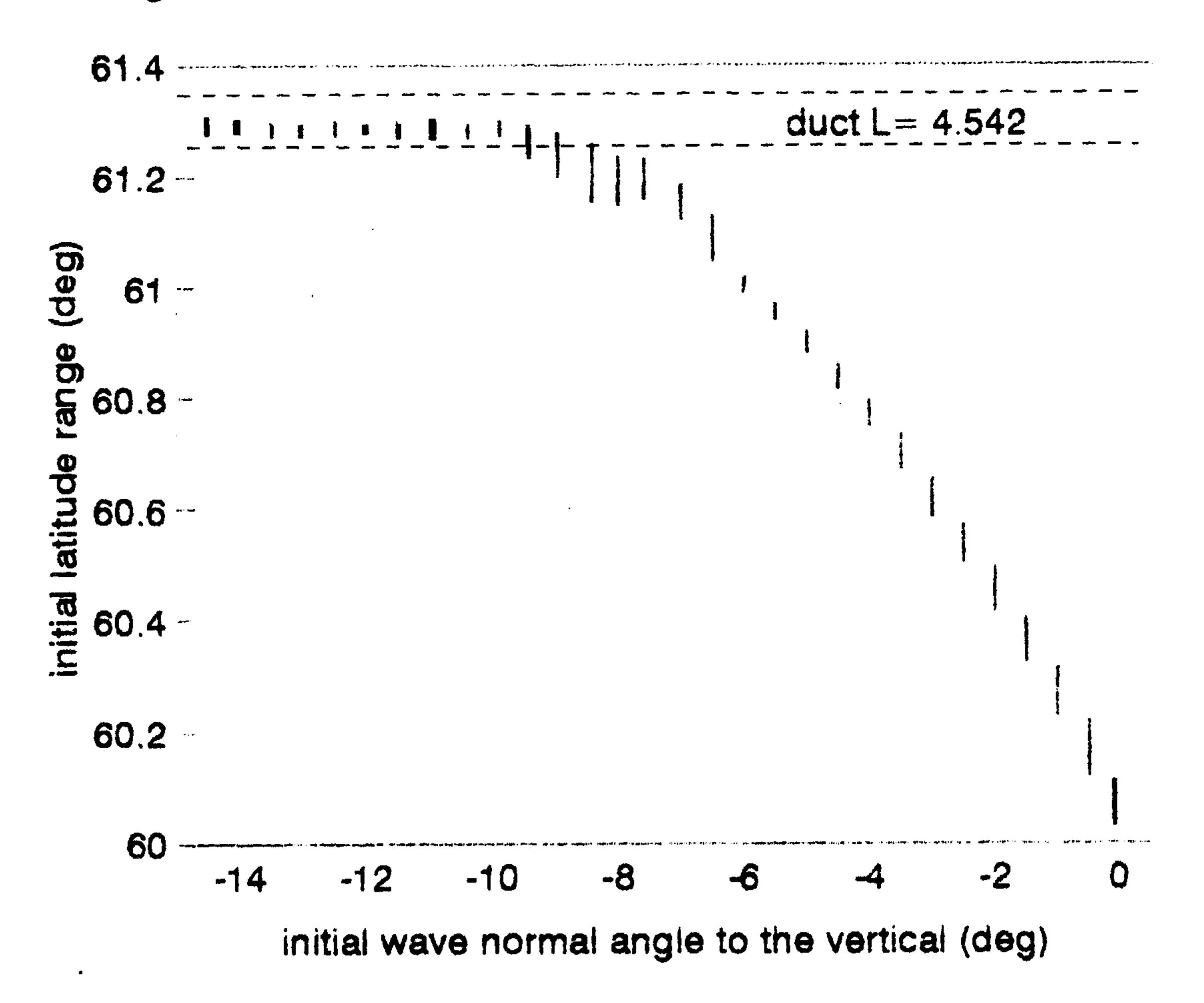


Fig. 3 Initial latitude range at 300 km altitude in the summer dawn hemisphere, plotted against initial direction of the wave normal for ducted 3 kHz ray.

Table 1

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dipole	dipole lat.	dipole lat.	final wave	height of	height of
latitude	of initial	of final	normal	enter point	exit point
of duct	range	range	angle	in km	in km
53.111°	52.710°	52.790°	-1° ~ 1°	800	850
	-52.630°	_52.302°			
	0.080°	0.488°	±1.15°	,	
54.747°	54.385°	54.499°	-1° ~ 1°	770	740
	1	-53.324°			
	0.064°	1.175°	±1.30°		
55.275°	54.922°	54.527°	-1° ~ 1°	1000	800
	]				
	0.061°	0.526°	±1.35°		
59.706°	59.161°	59.486°	-1°~1°	1200	1050
	-59.060°	_58.779°			
	0.101°	0.707°	士1.70°		
60.042°	59.406°	60.732°	-1°~1°	1250	1270
	-59.278°	-59.050°			
	0.128°	1.682°	士1.75°		
60.787°	59.850°	62.093°	-14° ~ -2°	1730	1600
		-59.941°			
	0.080°	2.152°	±1.80°		
61.025°	59.950°	61.914°	-13° ~2°	1900	1800
		-60.339°	-		
	0.129°	1.542°	±1.80°		
61.305°	60.122°	62.146°	-14° ~ -3°	1800	1700
	-60.016°	-60.794°			
	0.106°	1.352°	±1.80°		
	latitude of duct 53.111°  54.747°  59.706°  60.042°  61.025°	latitude of duct     of initial range       53.111°     52.710° -52.630° 0.080°       54.747°     54.385° -54.321° 0.064°       55.275°     54.922° -54.861° 0.061°       59.706°     59.161° -59.060° 0.101°       60.042°     59.406° -59.278° 0.128°       60.787°     59.850° -59.760° 0.080°       61.025°     59.950° -59.821° 0.129°       61.305°     60.122° -60.016°	latitude of duct         of initial range         of final range           53.111°         52.710°         52.790°           -52.630°         -52.302°           0.080°         0.488°           54.747°         54.385°         54.499°           -54.321°         -53.324°           0.064°         1.175°           55.275°         54.922°         54.527°           -54.861°         -54.527°           0.061°         0.526°           59.706°         59.161°         59.486°           -59.060°         -58.779°           0.101°         0.707°           60.042°         59.406°         60.732°           -59.278°         -59.050°           0.128°         1.682°           61.025°         59.850°         62.093°           -59.821°         -60.339°           0.129°         1.542°           61.305°         60.122°         62.146°           -60.016°         -60.794°	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$

for duct L = 4.542. The dashed lines show approximate extend of duct at 300 km altitude. Inclination is in the range -15.3° - 0°. The initial latitude range is inside duct base when the wave normal is parallel with geomagnetic field direction. As wave normal direction moves from geomagnetic field direction to vertical direction the initial latitude range moves equatorward from duct center for  $\sim 1^\circ$ .

Important effect on ducted whistlers is the variation of phase refractive index along the length of a duct (Smith, 1960; Helliwell, 1965). In the magnetosphere the refractive index for whistler is

$$\mu^2 \cong f_p^2/(ff_c \cos \psi - f^2) \tag{2}$$

where:  $f_p$  is plasma frequency;  $f_c$  is cyclotron frequency and f is wave frequency. The angle between wave normal and geomagnetic field is denoted by  $\psi$ . The refractive index is directly proportional to electron density and inversely proportional to geomagnetic field strength. Inhomogeneity in the magnetosphere curves a ray path and determines conditions for refraction.

For schematic presentation of the ray path in a meridian cross section magnetosphere, the results of ray-tracing 3 kHz ray are given on the Figure 4. Ray path in the duct L = 3.143 and the duct center position are given with the full lines. Dashed lines correspond to the boundary positions of the duct. The ray path is plotted versus distance along the field line. The ordinate is plotted in L value so the geomagnetic field lines are straight horizontal lines. Vertical dotted line corresponds to the projection of the equatorial plane. The left side of the Figure 4 represents summer dawn hemisphere and the right side winter night hemisphere.

The ray was trapped in the duct through the side of the duct at 770 km altitude. The electron density along the field line falls off rapidly and there is also decrease of the cyclotron frequency along the duct. Consequently, the minimum of the refractive index occurs at 750 km and after that increases with altitude. The refractive index at the equatorial region appears to be equal for both summer and winter hemispheres, giving the necessary continuity in this

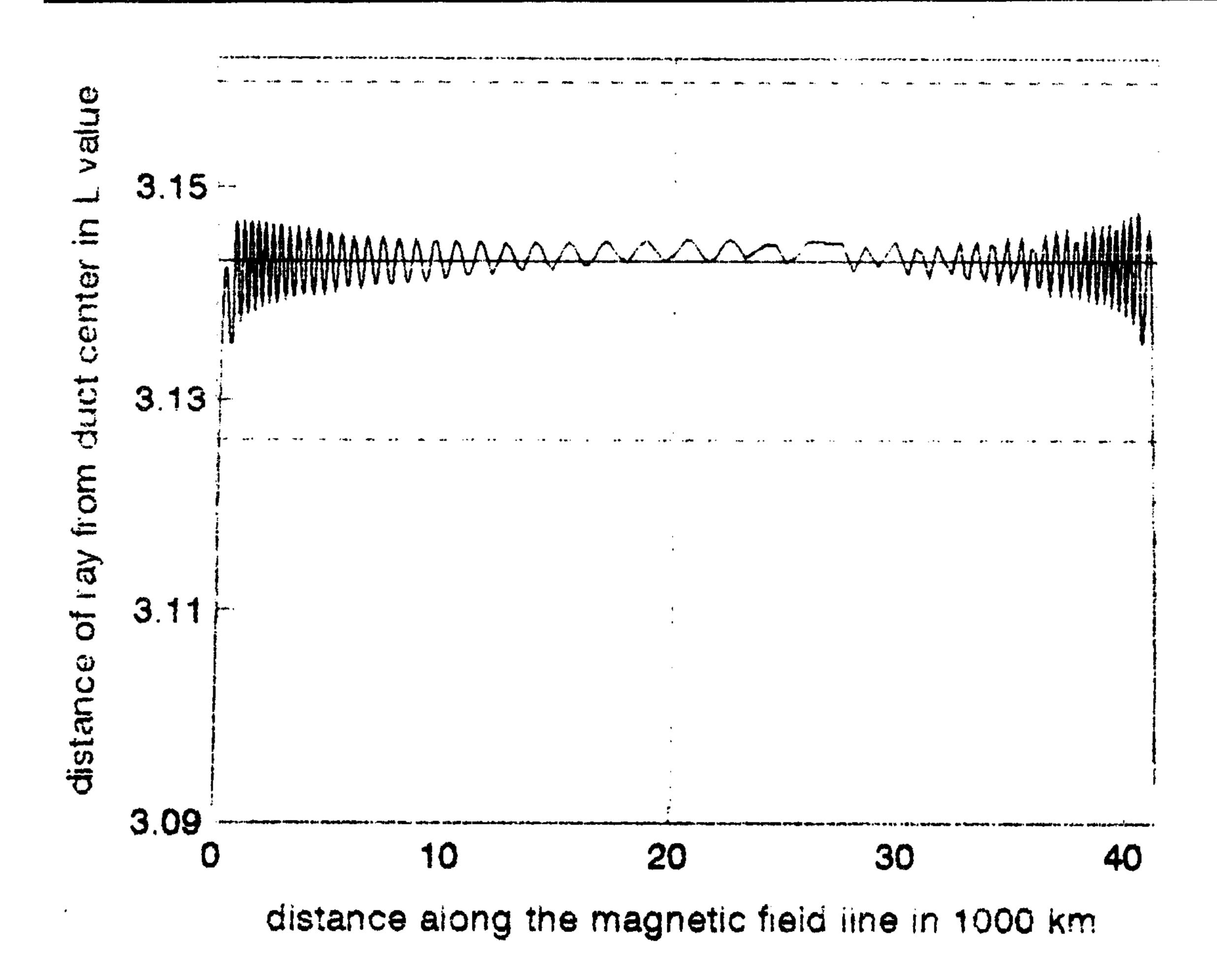


Fig 4 Ray path of 3 kHz ray started at  $54.037^{\circ}$ N dipole latitude in the summer dawn hemisphere. The ray is trapped through the side of 15% Gaussian duct, centered at L = 3.143 and propagates into the winter night hemisphere and escapes through the side of the duct.

parameter (Strangeways, 1982; Sulic, 1993). From equatorial plane the refractive index decreases with decreasing altitude having minimum value at 740 km altitude in the winter night hemisphere. The ray path excursions from the duct center decrease as the ray approaches the equatorial plane, due to the increasing refractive index along the length of the duct. After passing the equatorial plane the ray path excursions increase as the refractive index decreases along the duct in the winter hemisphere. Around the altitude of the minimum value of the refractive index ray escapes through the side of the duct.

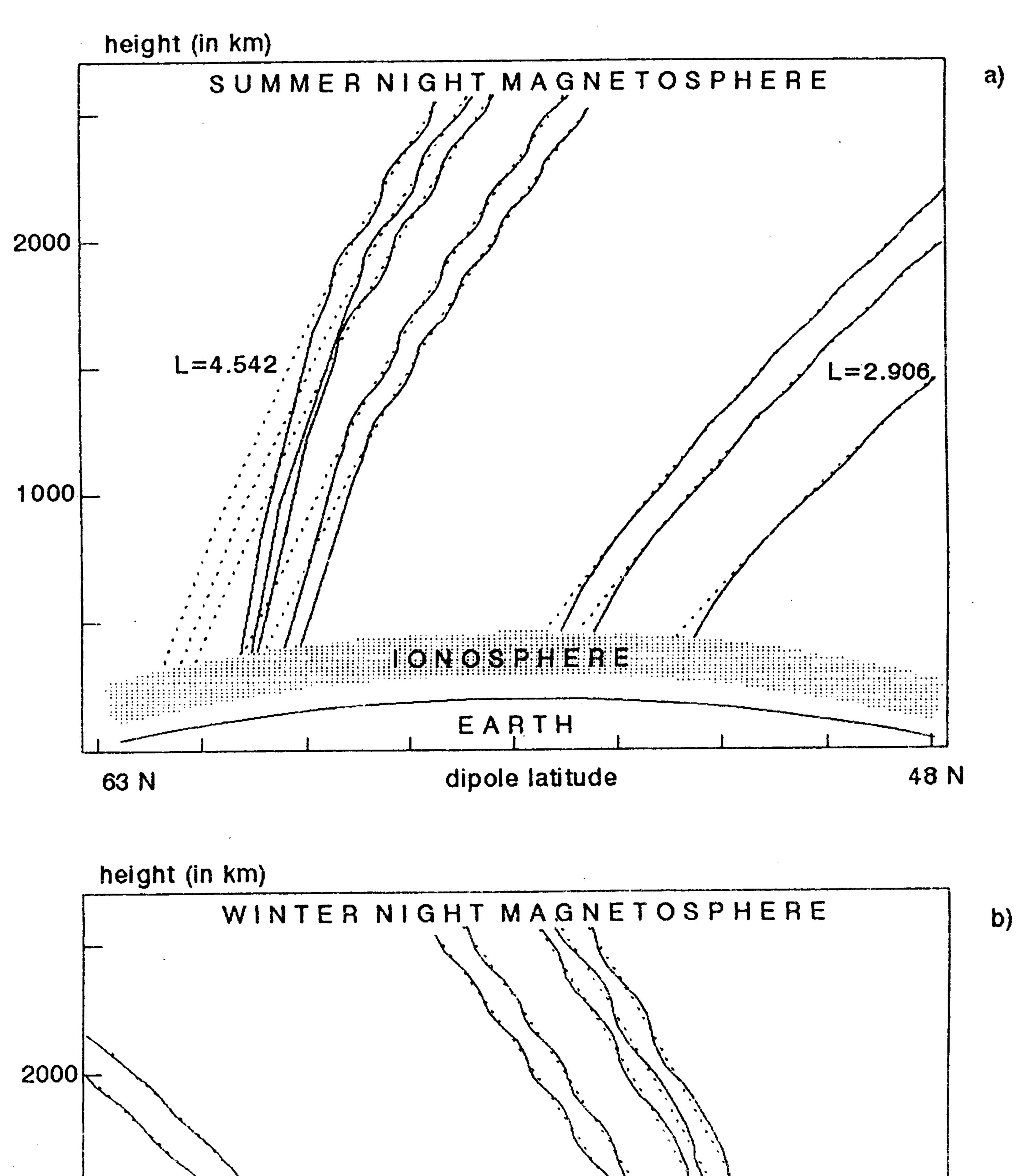
Figure 5 schematically shows positions for enter and exit points for 3 kHz rays that were trapped in all 8 duct. Figures 5a and 5b represent the duct center positions (dotted lines) and ray paths (full lines) in the function of the dipole latitude for summer dawn (northern) and winter night (southern) hemisphere, respectively. On the ordinate height till 2500 km is plotted, because the full electron density enhancement is reached at 2100 km height. From initial positions upgoing rays propagate in unducted mode up to heights where they become trapped through the side of the ducts. The heights where whistlers become trapped have tendency of increase with increasing L value of the duct. The positions in the magnetosphere for trapping whistlers are defined with the minimum values of the refractive index. After the rays are trapped in the summer hemisphere they propagate in ducted mode to the winter hemisphere. Continuous change of the refractive index in the equatorial plane causes the downcoming ray to propagate in the winter hemisphere. In the regions

of the minimum value of the refractive index in the winter hemisphere the rays escape through the side of the ducts.

#### 5. SUMMARY OF THE RESULTS

The trapping of VLF wave energy into and out of a duct may depend on electron density enhancement, width of duct, how low the duct extends downward in altitude and on altitude at which the full enhancement is reached. It is very difficult to measure all these parameters by satellite and compare with whistler data. The ray-tracing method is very useful in prediction of duct features in various models of the magnetosphere and to estimate the initial conditions for ray to be trapped and remained in the duct. The main results are:

- i Initial latitude ranges have average value 0.1° (that corresponds to distance of 12 km). The 3 kHz ray is trapped, when it starts from 0.4° to 1.2° equatorward from duct centers.
- ii Final latitude ranges are greater than initial latitude ranges and can have value as 2°. In the winter night hemisphere final latitude ranges are spread around duct centers, for ducts with L > 4.4 value.
- iii For ducts at lower L values final wave normal angles to the vertical lie inside the transmission cone. On contrary for ducts with L > 4.4 value all wave normal angles to the vertical lie outside the transmission cone.



2000

L=2.906

1000

L=4.542

1000

EARTH

48 S dipole latitude 63 S

Fig. 5 Ray paths of 3 kHz ray from 300 km to 2500 km height in the summer dawn and winter night hemispheres.

- iv Changing the direction of the wave normal from the magnetic field direction to the vertical direction the initial latitude range moves ~ 1° equatorward from the base of the duct.
- v In all analyzed cases the rays are trapped in the duct through the side of the duct in the summer-dawn hemisphere and escaped out through the side of the duct in the winternight hemisphere. Altitudes for trapping and escaping VLF waves from the duct are similar.

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# RAY-TRACING ИЗУЧАВАЊА ИНИЦИЈАЛНИХ УСЛОВА У АСИМЕТРИЧНОМ МОДЕЛУ МАГНЕТОСФЕРЕ ЗА ВИШЕ-КОМПОНЕНТНЕ ВИСТЛЕРЕ

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УДК 523.31—854:52—77\*3kHz Оригинални научни рад

Приказани су резултати израчунавања гауtracing методом иницијалних и коначних опсега ширина за сигнал од 3 kHz, који је заробљен у таласоводима. Осам таласовода је инкорпорирано од L > 2.9 до L < 4.6, у модел магнетосфере за услове лето-зима. За таласе који се емитују са хемисфере у условима лета и у току локалне зоре, иницијални опсези ширина имају средњу вредност 0.1° и померени су ка екватору у односу на центар таласовода. За надоле силазеће таласе у зимској и ноћној хемисфери коначни опсези ширина,

до којих ови доспевају, имају веће вредности него иницијални опсези ширина. Таласоводи лоцирани на L > 4.4 имају опсеге коначних ширина распрострањене око центара таласовода. Заробљавање таласа, који се простиру у моду вистлера, одвија се кроз омотач таласовода. Надоле силазећи талас напушта таласовод такође кроз омотач. Минимална вредност индекса рефракције дуж таласовода проузрокује заробљавање VLF таласа и њихово напуштање таласовода кроз омотач.