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ANALYSIS OF THE SYSTEMATIC EFFECTS AFFECTING (O – C) DETERMINATION OF THE OUTER PLANETS OBSERVED WITH THE BELGRADE VERTICAL CIRCLE

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SUMMARY: In this paper we show that the method used in the (O – C) calculation for the outer planets cannot entirely eliminate the systematic influences: time factor, temperature, flexure and refraction.The corrected (O – C) values for these influences as well as for the (E – W) effect are given.

1. INTRODUCTION

The meridian observations of the outer planets with the Belgrade Vertical Circle (BVC) have been regularly carried out from 1983.Two observers take part in the observations: V. Trajkovska and Dj. Bozhichkovich.Her own observations for the periods 1983 - 1984 and 1985 - 1986 V. Trajkovska published in "Bulletin de l'Observatoire Astronomique de Belgrade" (Trajkovska, 1986; Trajkovska, 1988).

In this paper an analysis of the unaccounted remaining systematic effects which appear in $(O - C)$ determinations of the outer planets with BVC is given.For this analysis we use the observations performed by V. Trajkovska in July 1984 (the first period) and July 1985 (the second period), because in these months the number of observational nights was sufficiently large, as well as those of the observed stars and planets.

2. THE METHOD OF (O–C) DETERMINATION OF THE PLANETS

In the $(O - C)$ determination (more precisely $\delta_O - \delta_C$) of the outer planets the method, applied in the treatment of the meridian planet observations performed with the Vertical Circle of Kiev Observatory (Kharin, et al., 1980), was used.

Since all the observations were carried out south of zenith ($\varphi \approx +45°$), the declinations of the planets δ_O are determined according to the formula $\delta_O = \varphi_0 - z_p$, where φ_0 is the latitude of BVC for a given observational night and z_p is the zenith distance of the planet.The latitude φ_0 was obtained from the observations of fundamental stars from the FK4 Catalogue.According to the formula $\varphi = \delta + z$ the latitude is obtained from the observation of each star, and for φ_0 the mean value $\overline{\varphi}$ is taken as derived from the observations of all stars.

In order to account to the maximum possible degree for the systematic influences affecting $\overline{\varphi}$ for the purpose of its determination are used stars whose coordinates are as close as possible to those of the planets. Sometimes in order to fullfil this condition as rigorously as possible for the purpose of the $\overline{\varphi}$ determination only those stars of the observatyionsal series are used whose coordinates are close to those of the corresponding planet.However, due to the lack of suitable stars this condition could not be fulfilled in all cases.

The zenith distances z are calculated according to the formula given earlier (Trajkovska, 1986).

The ephemeris of the planets (topocentric coordinates) δ_C for 1984 are obtained from the Pulkovo Observatory, and for 1985 from the Institute of Theoretical Astronomy, Leningrad.

3. THE GENERAL DATA

In Table 1 the general data of the observations are given.The observational nights, where the number of observed stars is less than six, we do not take into account, because, in our opinion, they do not satisfy the requirements of our analysis.In the second period there are three such nights.Regardless of this the number of observational nights, as well as that of observed stars and planets, is sufficient to admit a comprehensive analysis.

Since the observations with BVC are carried out at both clamps, in Table 1 are given the data on the initial instrument clamp.The temperature is expressed in centigrades, the pressure in mb.

Table 1. General Data

Date	Numbers of stars	Clamp E	Clamp W	Numbers of planets	Clamp E	Clamp W	Temperature	Pressure
1984 07 02	9	4	5	2	–	2	23.9–19.5	984.0
10	9	5	4	4	1	3	20.9–18.8	992.7
11	11	5	6	3	1	2	22.4–20.0	992.6
12	12	5	7	4	3	1	24.0–25.4	992.7
13	10	5	5	3	2	1	27.0–25.7	989.2
14	9	3	6	4	3	1	24.9–22.0	986.8
15	6	2	4	3	2	1	25.6–24.4	978.8
19	10	5	5	4	2	2	17.2–15.5	990.8
20	10	5	5	3	1	2	17.4–15.8	989.8
21	7	4	3	3	1	2	18.3–17.8	990.3
22	11	5	6	4	2	2	20.2–19.0	991.4
1985 07 04	10	4	6	3	3	–	16.0–13.6	994.3
05	17	8	9	3	3	–	16.8–14.7	989.2
06	8	3	5	2	2	–	17.0–15.4	990.0
15	9	5	4	2	1	1	20.9–19.7	992.7
16	18	10	8	3	–	3	22.8–20.9	993.8
22	17	10	7	3	–	3	18.0–15.5	998.4
25	7	2	5	1	1	–	20.9–18.7	990.7
29	9	6	3	2	–	2	26.1–29.3	983.8
30	8	5	3	1	–	1	26.0–27.1	984.2

4. ANALYSIS

The values $(O-C) = \delta_O - \delta_C$ are infested by accidental and systematic errors of the determination of δ_O, as well as by systematic errors in δ_C appearing as a consequence of insufficient accuracy of the planetary and the Eart's orbital elements and of the theory describing their motion.Since the meridian observations of the planets, together with the photographic and others, have been carried out exactly for the purpose of correcting their orbital elements, in this paper we shall consider only the systematic influences affecting the δ_O determination.

As the observed declinations of the planets $\delta = \overline{\varphi} - z_p$ are burdened by systematic errors of the determinations of $\overline{\varphi}$ and z_p, we shall consider each of these cases separately.

4.1 Systematic Errors in the $\overline{\varphi}$ Determination

A preliminary analysis of the φ values obtained from observations of the stars during the observational night showed, that a dependence of these quantity on both coordinates of the observed stars existed.The dependence on α (duration of the observations) is correlated with temperature which on the whose droped during the observations.Therefore, this dependence may be due to a combined effect of the time factor and temperature.The dependence on δ may be, most likely, a consequence of unaccounted effects of refraction since the observations were performed at relatively high zenith distances. In the treatment of observations of stars and planets the flexure is not included, whereas the refraction is calculated according to the Pulkovo Tables of Refraction (fourth edition).

If the existence of unaccounted systematic influences affecting the zenith-distance determinations due to a linear influence in α, flexure and refraction is assumed, then φ_i obtained from the observations of stars can be represented by the following formula:

$$\varphi_i = \varphi_0 + a(\alpha_i - \alpha_0) + b \sin z_i + \Delta k \tan z_i \qquad (1)$$

where

φ_0 – BVC latitude for observational night;
a – coefficient of dependence on α;
$\alpha_0 = \sum \alpha_i / n$;
b – coefficient of the horizontal flexure component;
Δk – coefficient of the refraction correction.

According to (1) the mean value $\overline{\varphi}$ is:

$$\overline{\varphi} = \varphi_0 + b\overline{\sin z_i} + \Delta k \overline{\tan z_i} \qquad (2)$$

According to (2) if for the determination of δ_O one takes $\overline{\varphi}$ in addition to the required value φ_0, one accounts also the part of the unnacounted effects for the flexure relating to the zenith distance z' ($\sin z' = \overline{\sin z_i}$) and for the refraction relating to the zenith distance z"($\tan z$" $= \overline{\tan z_i}$). These zenith distances, in our case, are close to the mean zenith distance of observed stars $z = \sum z_i / n$.

On the Tenth National Conference of Yugoslav Astronomers held in Belgrade (Yugoslavia), we reported the preliminary results of an analysis concerning the unaccounted systematic effects of the flexure and of the refraction (Trajkovska and Mijatov, 1993).For this analysis an analogous formula is used:

$$\varphi_i = \varphi_0 + b \sin z_i + \Delta k \tan z_i \qquad (3)$$

The mean latitude $\overline{\varphi}$ from (3) is identical with $\overline{\varphi}$ from (2), and by its application are accounted the effects of the flexure and refraction for the zenith distances z' and z".

Since it is of interest to answer the question of how much the effects of α can influence $(O - C)$, we decided to apply (3), taking into account, that in the mentioned report we gave only the results concerning the $(O - C)$ corrections of the planets for the first period.

If we denote the published $(O - C)$ values obtained by applying the mean latitude $\overline{\varphi}$ as $(O - C)_{\overline{\varphi}}$, and those obtained by applying φ_p from (1) and (3) for the planet zenith distances as $(O - C)_{\varphi_p}$, then the differences $\Delta(O - C) = (O - C)_{\varphi_p} - (O - C)_{\overline{\varphi}}$ are corrections of the published values $(O - C)$ assuming that there are unaccounted influences due to the linear change in α, flexure and refraction or only in the two latter ones as given in (3).

As the values are $(O - C)_{\overline{\varphi}} = \overline{\varphi} - z_p - \delta_C$ and $(O - C)_{\varphi_p} = \varphi_p - z_p - \delta_C$ then we have

$$\Delta(O - C) = \varphi_p - \overline{\varphi} \qquad (4)$$

The correction $\Delta(O - C)$ is equal to the difference of the applied latitudes φ_p and $\overline{\varphi}$.

On the basis of (1), (2), (3) and (4) it is

$$\Delta(O - C)_1 =$$

$$= a(\alpha_p - \alpha_0) + b(\sin z_p - \overline{\sin z_i}) + \Delta k(\tan z_p - \overline{\tan z_i}) \qquad (5)$$

$$\Delta(O - C)_2 =$$

$$= b(\sin z_p - \overline{\sin z_i}) + \Delta k(\tan z_p - \overline{\tan z_i}) \qquad (6)$$

Therefore, in order to calculate the $\Delta(O - C)_{1,2}$ values it is necessary to determine from (1) and (3) the unknown quantities a, b and Δk. The common characteristics of these systems are the following. Since the observations of stars, in accordance with the method, were performed within narrow zones, up to 15° in declination, the total changes in $\sin z$ are small, but those in $\tan z$ much larger. The differences of the coefficients of the unknowns φ and b are approximately equal due to the small changes in $\sin z$ so that these unknowns cannot be satisfactorilly separated. The total effect of $b \sin z + \Delta k \tan z$ affecting the quantity $\Delta(O - C)$ is practically the same for different values of φ_0.

On account of the mentioned characteristics of the systems we decided to use for the mean latitude of BVC the value $\varphi_0 = +44°48'07''.60$. This value was obtained for the period 1983 – 1985 in addition to other parameters from a comparison of declinations of 212 FK4 stars observed with BVC and with corresponding declinations in the FK4 System (Bozhichkovich, 1991).

In Table 2 are given the mean values of the coefficients a, b and Δk with their errors obtained from (1) and (3) for the observations of the first and second periods. In addition to them we also present the mean values of σ, the errors of standard deviation of the corresponding equations, and $\varepsilon_{\overline{\varphi}}$, the error of $\overline{\varphi}$.It should be noted that for the determination of the quantities from Table 2 relating to the observations in the two periods in both cases the data for one observational night are omitted because of extremal values of b and Δk compared to other corresponding values. From the relation (3) the coefficients a are determined from the residuals obtained after applying the values b and Δk for the corresponding observational night.

Table 2. The Values a, b, Δk, σ and $\varepsilon_{\overline{\varphi}}$

Period	Expression	a	b	Δk	σ	$\varepsilon_{\overline{\varphi}}$
July	(1)	$-0''.01\pm 0''.10$	$-0''.48\pm 1''.72$	$+0''.46\pm 0''.67$	$0''.45$	$0''.46$
1984	(3)	-0.06 ± 0.13	-0.48 ± 1.60	$+0.45\pm 0.63$	0.45	0.46
July	(1)	-0.03 ± 0.10	-1.92 ± 1.66	$+0.88\pm 0.69$	0.54	0.56
1985	(3)	-0.10 ± 0.13	-1.83 ± 1.58	$+0.78\pm 0.65$	0.54	0.56

The existence of practically the same dependence on α for both periods in the equations (1) and (3) of Table 2 indicates possible reality of this influence though the values in (1) are small. The differences between (1) and (3) arise most likely as a result of the treatment according to the least-square method because in (1) for the same, small, number of equations of condition the number of unknowns is increased. However, the values b and Δk due to the mentioned system characteristics and very different individual values cannot be considered as realistic, but the sum $b \sin z + \Delta k \tan z$ can be considered as such. In favour of this conclusion is also the fact that $(O-C)_{\varphi_p}$, as already mentioned, does not depend on φ_0 in both (1) and (3), while by variation of φ_0 the individual values of b and Δk in these expressions are changed.

Since the values of a and $b \sin z + \Delta k \tan z$ can be considered as realistic, it follows that for every observational night realistic values for $\Delta(O-C)_1$ and $\Delta(O-C)_2$ can be obtained from (5), i. e. (6) for each of the observed planets.

Table 3 contains the mean values of $\Delta(O-C)$ of the planets for (1) and (3) including the observations of both periods. In the parentheses is given the number of observations.

Table 3. The Mean Values of $\Delta(O-C)$ of the Planets

Period	Expression	Mars	Jupiter	Saturn	Uranus	Neptune
July	(1)	$-0''.09$ (9)	$+0''.01$ (8)	$-0''.25$ (1)	$-0''.01$ (10)	$-0''.04$ (9)
1984	(3)	-0.15 (9)	$+0.07$ (8)	-0.20 (1)	-0.04 (10)	-0.02 (9)
July	(1)	–	-0.05 (3)	-0.12 (3)	$+0.11$ (8)	$+0.03$ (6)
1985	(3)	–	-0.08 (3)	-0.13 (3)	$+0.09$ (8)	$+0.06$ (6)

The individual values of $\Delta(O-C)$ can be significant, even greater than $0''.4$. More than 10% of all the values are greater than $0''.2$ and over 30% greater than $0''.1$. In most cases the mean values contained in Table 3, also, cannot be considered as negligible.

The differences between (1) and (3) cannot be considered as negligible, especially for the cases of Mars and Jupiter in the first period. It should be mentioned that α of these planets were in the case of the former smaller and in the case of the latter larger than α of almost all observed stars and consequently significantly farther from α_0. It follows from Table 3 that the dependence on α can influence considerably the amount of $\Delta(O-C)$ if the α values of the planets are considerably different from α_0. Based on (3), where $\Delta(O-C)$ depends on δ of stars and planets only, the same conclusion can be also drawn for δ. The declinations of Mars in the first period and of Saturn in both were higher than δ of all observed stars, whereas for Jupiter in the first and Uranus in the second period they were lower than those of almost all stars observed.

Based on (1) and (3) one can conclude that in our case the effect of δ on $\Delta(O-C)$ exceeded the one of α and that in most cases the effects of the two influences were opposite, i. e. the δ effect was toned down by that of α.

All said above indicates that the method applied requires an exceptional rigorousness when the planet coordinates are close to the mean ones of the stars. The rigorousness required by the method cannot in praxis, in most cases, be achieved, even when a selective choice of stars is done.

4.2 The Systematic Errors in the Determination of z

As already said in Section 2, every observation of stars and planets with BVC has been performed at both clamps so that in this way the influences of a certain number of instrument as systematic errors are diminished. Hence we shall consider here only the joint influence of unaccounted systematic errors manifested through the effect $(E-W)$. This effect is due to the possible systematic differences in the measured zenith distances connected with the initial clamp and it can be determined for an observational night from the differences $\varphi_E - \varphi_W = z_E - z_W$ for the stars observed. It does not affect the $\overline{\varphi}$ values if one neglects the unequal number of star observations for the initial clamps E and W during the observational night, but it affects the z determination of the planets.

Table 4. The $(E - W)$ Values for Observational Nights

Date	E–W	Date	E–W
1984 07 02	+0.″16	1984 07 22	+0.″05
10	0.03	1985 07 04	−0.01
11	−0.08	05	−0.28
12	+0.30	06	−0.04
13	−0.05	15	−0.10
14	+0.27	16	+0.18
15	0.00	22	−0.05
19	−0.24	25	−0.36
20	−0.34	29	−0.61
21	−0.02	30	−0.38

Table 4 contains the mean values of the $(E - W)$ effect, observational order EW, i. e. WE for the stars, for the observations performed during the first and second periods. The number of observed stars at the initial clamps E and W is given in Table 1.

As seen from Table 4 the $(E - W)$ values for some dates are considerable.

Since $\overline{\varphi}$ is free of the $E - W$ effect, it is necessary to free also the zenith distances of the planets from the same effect and thereby also the published $(O - C)$. If $\varphi_E - \varphi_W = \Delta z$, then the values of the planets zenith distances corrected for this effect are equal to $z_E - \Delta z/2$, i. e. $z_W + \Delta z/2$, depending on the clamp respectively. Based on the values $(E - W)$ from Table 4 the corrected values of the planets zenith distances can be calculated and therewith also the corrections of the published $(O - C)$ values.

The effect $(E - W)$ influences on the $(O - C)\overline{\varphi}$ values only, but not on the $\Delta(O - C)_{\varphi_p}$ because $\overline{\varphi}$ and φ_p are free of this effect.

In Table 5 are given the mean $(E - W)$ values representing the corrections of the $(O - C)$ values for the planets for this effect. The number of observations is given in the parentheses.

Table 5. The Influence of the Mean Values of the $(E - W)$ Effect on the Planets $(O - C)$

Period	Mars	Jupiter	Saturn	Uranus	Neptune
July 1984	+0.″01 (9)	0.″00 (8)	+0.″08 (1)	+0.″06 (10)	−0.″06 (9)
July 1985	–	+0.08 (3)	+0.08 (3)	−0.04 (8)	−0.02 (6)

The mean values of the $(E - W)$ effect given in Table 5 merely illustrate the fact already shown in Table 4 that this effect must be taken into account for most of the observational nights and that the published $(O - C)$ values have to be corrected for it.

Therefore, the published $(O - C)$ values obtained for every observation of the planets should be corrected for two influences - because α and δ of the planets are not sufficiently close to the mean values α_0 and δ_0 of the observed stars and the influence of the $(E - W)$ effect. In some cases the application of these corrections can improve the published $(O - C)$ values considerably.

5. CONCLUSION

This analysis shows that the method applied for the determination of the $(O - C)$ values of the planets with BVC cannot entirely eliminate the remaining unaccounted systematic effects affecting these values.These effects, as shown, are due to a combination of influences connected with the duration of the observations, the change of the temperature, of the flexure, and of the refraction, so that their total influence caused, in some cases, considerable systematic errors in the determination of the $(O - C)$ values.According to the used method the unacounted effects are negligible only when the coordinates of planets were very close to the mean coordinates of the observed stars.Since this condition was very difficult to fulfill in practice, the $(O - C)^s$ were determined with systematic errors whose values were dependent on to what measure this condition was satisfied.

In addition to the mentioned cause of the systematic errors in the determination of the planets' $(O - C)$ there is the considerable influence of the neglected $(E - W)$ effect.Due to this effect corrections of the published values are determined and with their application this effect can be eliminated.Of course in the future reductions this effect must certainly be taken into account.

The disadvantage of the used method is compensated for by the presented procedure of determination of unaccounted influences, which are due to the mentioned dependence on α and δ, which makes possible the determination of the corrections to the $(O - C)$ values.

However, since, the used method does not make possible obtaining the $(O - C)$ values without systematic errors it is necessary to consider the question whether the used procedure for the determination of systematic effects is sufficient that it can be accepted in the future as a method for reduction of the observations of planets with BVC, or it is necessary to look for a completely new method.

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АНАЛИЗА СИСТЕМАТСКИХ ЕФЕКАТА КОЈИ УТИЧУ НА ОДРЕЂИВАЊЕ (О – С) СПОЉНИХ ПЛАНЕТА ПОСМАТРАНИХ НА БЕОГРАДСКОМ ВЕРТИКАЛНОМ КРУГУ

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Оригинални научни рад

У овом раду је показано да примењена метода за одређивање $(O-C)$ спољних планета не може у потпуности да елиминише систематске утицаје везане за временски фактор, температуру, савијање и рефракцију. Дате су поправљене вредности $(O-C)$ за ове утицаје, као и за ефекат $(E-W)$.