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Preliminary report

SYSTEMATIC ERRORS OF SOME PARAMETERS WHICH HAVE BEEN USED IN THE TREATMENT OF BELGRADE CATALOGUE OF ABSOLUTE DECLINATIONS (BCAD)

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SUMMARY: The systematic errors of the latitude, corrections of the refraction and flexure – values, which have been applied in the treatment of BCAD, are given in this paper. They have been obtained by comparison of BCAD and FK5. It was found out that the mean systematic errors are $+0''.14$, $-0''.02$ and $+0''.14$ respectively for the mentioned parameters and that there exists significant seasonal variations of the refraction corrections in winter and flexure in autumn and winter.

1. INTRODUCTION

The analysis of the systematic differences $\Delta\delta_\alpha$ and $\Delta\delta_\delta$ of the Belgrade Catalogue of Absolute Declinations (BCAD) related to fundamental catalogues FK4, FK5, and GC have shown that this catalogue is overloaded with systematic errors type $\Delta\delta_\alpha$ and $\Delta\delta_\delta$ (Mijatov et al. 1992). It was found that systematic errors $\Delta\delta_\delta$ are the result of the existing significant systematic influences in the zones of 2° of declination. It was supposed that these influences are the result of the systematic errors in the determination of corrections to the circle readings. On the other hand the systematic errors in the values of refraction and flexure are also a source of the systematic errors $\Delta\delta_\delta$ and $\Delta\delta_\alpha$. The last ones depend on the errors which have a seasonal character.

In this paper the systematic errors of BCAD depending on the inaccurate determination of latitude refraction and flexure are analysed. Determination of the reasons for existance of systematic errors in the zones of 2° of declination due to its character related to the determination of latitude refraction and flexure, will be the object of the special analysis.

2. SYSTEMATIC ERRORS RESULTED FROM INACCURACY OF DETERMINATION OF LATITUDE, REFRACTION AND FLEXURE

All stars of BCAD are observed by the absolute method in both culminations on Belgrade Vertical Circle (BVC).

The zenith distances of the stars are calculated by the following relation.

$$z = z' + \rho - b\sin z + \Delta z' + \Delta z'' + \Delta R \tan z \quad (1)$$

where are

z' – apparent zenith distance;

ρ – refraction;

$b\sin z$ – the first member of the whole flexure $b\sin z + b'\sin 2z + \ldots$;

$\Delta z'$ – corrections for systematic differences $z_{ew} - z_{we}$ determined for the zones of 5°;

$\Delta z''$ – corrections for systematic differences of zenith distances between the observers determined also for the zones of 5°;

ΔR – so - called correction of the refraction constant;

Refraction ρ is calculated according to the Pulkovo Refraction Tables (fifth edition).

Values b are calculated from the parameters obtained from analysis of collimator measurements of this value (Mijatov and Trajkovska, 1989), according to the relation:

$$b = b_0 + 0''.04(t - 12.9°C) + \Delta_1 + \Delta_2 + \Delta_3 \quad (2)$$

where the following notation is used:

b_0 – mean value of b for the mean observation temperature during the whole period;

$0''.04(t - 12.9°C)$ – temperature effect;

Δ_1 – dependence of b from the differences between observers;

Δ_2 – dependence from the evaluations of measurements of individual values b;

Δ_3 – dependence of diurnal variations of b.

The value ΔR is determined by the classical method, together with $\Delta\varphi_0$ (the correction of latitude φ_0) which is used for determination of the preliminary declinations in upper and lower culmination.

The possible influences on z which come from anomalous refraction $\rho' \sec^2 z$ and other terms of flexure are not taken into consideration.

Declinations of stars are calculated according to the relations:

$$\delta = \delta_1 + \frac{\delta_2 - \delta_1}{1 + a} \quad (3)$$

where the used notation is as follows:

δ_1, δ_2 – mean declinations obtained from observations in upper and lower culminations;

$$a = q(\frac{\varepsilon_2}{\varepsilon_1})^2$$

$\varepsilon_1, \varepsilon_2$ – accidental errors of δ_1 and δ_2;

$$q = \frac{(n_1 - 3)(n_2 - 1)}{(n_1 - 1)(n_2 - 3)}$$

n_1, n_2 – number of observations in upper and lower culmination.

Systematic errors of the determination of latitude, refraction and flexure – the main parameters in the treatment of BCAD – are the basic cause of the existance of its systematic errors of the type $\Delta\delta_\alpha$ and $\Delta\delta_\delta$.

Systematic errors in determination of z, which are the result of an inaccurate correction for refraction, in this case, could be a consequence of errors in determination of pure refraction ρ and negligence of anomalous refraction. Errors in the determination of ρ are the result of insufficient accuracy of applied refraction tables, as well as the systematic errors in the parameters for ρ, firstly temperature and preasure. Systematic errors which come from ρ could be significantly reduced by the application of correction $\Delta R \tan z$. However, if determined by applied method the value ΔR is not very reliable due to the existance of additional systematic influences (not only refractional ones).

Uncalculated effect which originates from anomalous refraction can be determined from aerological data. As it is known, for the determination of anomalous refraction, aerological data on the inclinations of atmospheric layers of equal density and refractional zenith azimuthes of these layers are necessary. In this case the ground layer is dominant, whilst the influence of the other layers is neglectable (Teleki, 1967).

The influence of anomalous refraction in the determination of z for the meridian observations can be obtained approximately from the following relation (Teleki, 1967):

$$\Delta z = \beta_0 \cos A_0 \ln n_0 \sec^2 z. \quad (4)$$

The applied notation is:

β_0 – inclination of the ground layer;

A_0 – azimuth of refractional zenith of this layer;

$n_0 = 1.000276$ – index of refraction for $\lambda = 0.5753, t = +15°C, P = 760$ mm Hg and $f = 4$ mm Hg;

Value $\beta_0 \cos A_0$ in the relation (4) can approximately be obtained from data for wind velocity and direction at different heights according to relation (Zverev, 1946):

$$\beta_0 \cos A_0 = -0'.063 \sin\varphi [8000 \frac{\Delta(V \sin\theta)}{\Delta h} - V \sin\theta] \quad (5)$$

with

φ – latitude of observational point;

V – wind velocity in m/sec;

θ – wind direction;

h – the height of the layer where V and θ were measured;

$\Delta(V \sin\theta)$ – difference $V_2 \sin\theta_2 - V_1 \sin\theta_1$ for two neighbouring layers;

Δh – height difference $h_2 - h_1$ of two neighbouring layers;

Close to the Belgrade Observatory, at a distance of 1.9 km and practically at the same altitude, an Aerological Observatory is situated. At this Observatory radiosonde measurements are being carried out daily (day and night). These measurements helped us to obtain necessary data for wind velocity and direction for ground layer ($P = 1000$ *mbar*) and $P = 850$ *mbar* layer. Using mean monthly values for V and θ for the period 1976 - 1980, from relation (5), for ground layer we obtained $\beta_0 \cos A_0 = +0'.9$. The value $+0'.5$ for $\beta_0 \cos A_0$ of the ground layer was reached by Teleki, (1967) using data from the same

Aerological Observatory but for the period 1955 - 1962.

Using the obtained value for $\beta_0 \cos A_0$ we can get Δz - correction for anomalous refraction $\Delta z = +0''.015 \sec^2 z$. In our case the maximum value for this effect on $\Delta\delta$ is $\Delta\delta = +0''.07$.

Systematic errors in determination of z which come from flexure, in our case, can appear because of the errors in the determination of term $b \sin z$, i.e. errors in determination of parameters which take part in the calculation of value b according to the relation (2) and negligence of other terms of flexure, especially $b' \sin 2z$.

If other systematic influences, including the influence which comes from systematic errors in 2° zone of declinations, are neglected, then the systematic errors of declinations of BCAD $\Delta\delta$ can be considered as the result of systematic errors in latitude refraction and flexure. After the elimination of anomalous refraction, these influences can be expressed together in the following way

$$(\frac{a-1}{a+1})\Delta\varphi + (\frac{a\tan z_1 - \cot z_1}{a+1})\Delta R' +$$

$$(\frac{\cos z_1 - a\sin z_1}{a+1})\Delta b = \Delta\delta - \Delta\delta' \qquad (6)$$

with

$\Delta\varphi$ - error of the adopted latitude φ;
$\Delta R'$ - inaccuracy in ΔR;
Δb - error of the mean value b;
$\Delta\delta = \delta_{BCAD} - \delta_{FK5}$
$\Delta\delta'$ - correction for anomalous refraction (since $\varphi \approx +45°$ we have taken $z_2 = 90° - z_1$).

The analysis has shown that in our case due to the character of the observation program and the treatment, a significant correlation between coefficients by $\Delta\varphi$ and b' exists, so by putting b' into the expression (6) this system would become insufficently strict. Therefore b' was missed out.

In Table 1 values $\Delta\varphi$, $\Delta R'$ and Δb with determination errors and standard deviation σ are given.

Table 1.

$\Delta\varphi$	$+0''.14$	$\pm0''.23$
ΔR'	-0.02	±0.22
Δb	$+0.18$	±0.71
σ	$\pm0''.25$	

Determination accuracy of unknown quantities is small, especially in the case of Δb. Determination accuracy of Δb is approximately 3 times smaller than determination accuracy of $\Delta\varphi$ and $\Delta R'$. This is the consequence of the use of a large number of parameters for calculation of b, and not of combining of the observations in upper and lower culmination even if they have been carried out in different seasons. Separating Δb for these two culminations gave approximately the same accuracy Δb_1 and Δb_2 as it is the accuracy of Δb. However, if for $\Delta\delta$ mean values for 2° zones are taken, the accuracy of unknown quantities increases for approximately 2.5 times, which proves that accuracy strongly depends on errors in determination of circle readings corrections.

Since the accuracy of unknown quantities is small it is necessary to consider the question of their reality.

The difference of an adopted latitude φ which was used in the treatment of BCAD and mean latitude φ_{zt} obtained at Zenith - telescope of Belgrade Observatory for the observation period of BCAD (and which was applied for the parallel of Vertical Circle) is $\Delta\varphi \approx +0''.15$. The value $\Delta\varphi$ from Table 1 approximately corresponds to this difference which proves its reality. On the basis of the proved dependence between $\Delta\varphi$ and b' and the reality of $\Delta\varphi$, it can be concluded that the second term of flexure, if it exists, is insignificant.

Concerning the reality of $\Delta R'$, it is difficult to make sure decision. However, $\Delta R'$ is the correction of ΔR, which is obtained from the observational data, it can be expected that this correction is small.

Certainly the most important question at this point, is the reality of Δb, because the flexure is the biggest problem of the Vertical Circles. If on the basis of the previously shown facts one takes that $\Delta\varphi$ and $\Delta R'$ are real as a hipothesis, then Δb should be also real.

The mean values of the parameters from expression (2) are: $b = +0''.47$; $b_0 = +0''.69$; $+0''.02(t - 12.9°C) = -0''.02$; $\Delta_1 = -0''.16$; $\Delta_2 = -0''.02$; $\Delta_3 = -0''.02$.

All applied corrections including a temperature effect result in a certain reduction of Δb, so they can be considered more or less real. The temperature coefficient $+0''.04$ was obtained also at other similar meridian instruments during flexure examination.

By eliminating the obtained systematic errors $\Delta\varphi$, $\Delta R'$ and Δb from $\Delta\delta$, the mean difference $\Delta\delta$ has been reduced from $+0''.05$ to $0''.02$, but the existing variations of $\Delta\delta$ in the limits of 2° declination zone have still remained, which means that the systematic influence which causes short - periodic variations $\Delta\delta$ is not eliminated nor reduced. This proves the previously mentioned statement that these variations are the result of systematic errors in determination of circle readings corrections.

3. THE ESTIMATION OF THE SEASONAL SYSTEMATIC ERRORS WHICH HAVE THEIR SOURCE IN THE INACCURACY OF THE REFRACTION AND FLEXURE DETERMINATION

By the analysis of systematic differences $\Delta\delta_\alpha$ of BCAD catalogue and fundamental catalogues FK4 and FK5, (Mijatov et al. 1992) proved that these differences are the result of systematic errors which came from uncalculated seasonal variations of refraction and flexure.

In Table 2 the values $\Delta\delta_\alpha$ in relation to FK5 are given. These values obtained by elimination of mean (1° declination) $\Delta\delta_5$ from $\Delta\delta$ taking into consideration the newly obtained corrections $\Delta\varphi$, $\Delta R'$ and Δb from Table 1. The mean values $\Delta\delta_\alpha$ are given in 4^h zones of right ascension and in the seasons in which the stars, which belong to these zones are observed. The first given season corresponds to observation in the upper culmination and the second one to the lower culmination. For the observation seasons have been taken these in which the majority of all observations have been done in the zone. N is the number of stars in the zone.

As can be seen from Table 2, the values $\Delta\delta_\alpha$ are small for the seasonal combination spring - summer but they are significant for the combinations summer - winter and spring - autumn. For the purpose of approximate estimation of the refraction and flexure influence on the seasonal variations $\Delta\delta_\alpha$, $\Delta R''$ (refraction influence) and Δb_1 and Δb_2 (flexure influence for the observations in upper and lower culminations) have been determined according to formula:

$$(\frac{a\tan z_1 - \cot z_1}{a+1})\Delta R'' - \frac{a\sin z_1}{a+1}\Delta b_1 + \frac{\cos z_1}{a+1}\Delta b_2 = \Delta\delta_\alpha - \Delta\delta' \qquad (7)$$

where $\Delta\delta'$ is correction for the seasonal influence which comes from anomalous refraction. The values $\Delta\delta'$ are determined from differences of seasonal parameters $\beta\cos A_0$ and the corresponding mean value $0\overset{''}{.}9$, which are $-0\overset{''}{.}3$, $-0\overset{''}{.}2$, $+0\overset{''}{.}1$ and $+0\overset{''}{.}4$ respectively for seasons spring, summer, autumn and winter.

In Table 3 the value $\Delta R''$, Δb_1, Δb_2 and standard deviation σ which are obtained by the method of least square from expression (7) for 4^h zones of right ascension are given.

Mean wieght values $\Delta R''$, Δb_1, Δb_2 from Table 3 are $0\overset{''}{.}00$, $-0\overset{''}{.}11$ and $0\overset{''}{.}02$ respectively. The values $\Delta R''$ and $\Delta(\Delta b) = \frac{1}{2}(\Delta b_1 + \Delta b_2)$ can be cosidered as corrections of systematic errors $\Delta R'$ and Δb, so according to this, $\Delta R'$ is well determined and Δb requires correction of $-0\overset{''}{.}04$.

Table 2.

α	$\Delta\delta_\alpha$	SEASONS	N
$0^h - 4^h$	$+0\overset{''}{.}02 \pm 0\overset{''}{.}03$	SUMMER - SPRING	38
4 - 8	$+0\overset{''}{.}17 \pm 0\overset{''}{.}04$	WINTER - SUMMER	38
8 - 12	$+0\overset{''}{.}01 \pm 0\overset{''}{.}04$	SPRING - SUMMER	39
12 - 16	$+0\overset{''}{.}08 \pm 0\overset{''}{.}03$	SPRING - AUTUMN	40
16 - 20	$-0\overset{''}{.}09 \pm 0\overset{''}{.}03$	SUMMER - WINTER	41
20 - 24	$-0\overset{''}{.}03 \pm 0\overset{''}{.}03$	SUMMER - SPRING	40

Table 3.

α	$\Delta R''$	Δb_1	Δb_2	σ
$0^h - 4^h$	$+0\overset{''}{.}18 \pm 0\overset{''}{.}23$	$+0\overset{''}{.}05 \pm 0\overset{''}{.}32$	$+0\overset{''}{.}30 \pm 0\overset{''}{.}47$	$\pm 0\overset{''}{.}20$
4 - 8	$-0\overset{''}{.}24 \pm 0\overset{''}{.}18$	$-0\overset{''}{.}77 \pm 0\overset{''}{.}23$	$-0\overset{''}{.}34 \pm 0\overset{''}{.}34$	$\pm 0\overset{''}{.}22$
8 - 12	$-0\overset{''}{.}03 \pm 0\overset{''}{.}16$	$-0\overset{''}{.}20 \pm 0\overset{''}{.}20$	$-0\overset{''}{.}17 \pm 0\overset{''}{.}30$	$\pm 0\overset{''}{.}21$
12 - 16	$+0\overset{''}{.}22 \pm 0\overset{''}{.}22$	$+0\overset{''}{.}31 \pm 0\overset{''}{.}26$	$+0\overset{''}{.}78 \pm 0\overset{''}{.}42$	$\pm 0\overset{''}{.}23$
16 - 20	$-0\overset{''}{.}21 \pm 0\overset{''}{.}18$	$-0\overset{''}{.}07 \pm 0\overset{''}{.}24$	$-0\overset{''}{.}50 \pm 0\overset{''}{.}37$	$\pm 0\overset{''}{.}21$
20 - 24	$+0\overset{''}{.}08 \pm 0\overset{''}{.}13$	$+0\overset{''}{.}02 \pm 0\overset{''}{.}20$	$+0\overset{''}{.}05 \pm 0\overset{''}{.}26$	$\pm 0\overset{''}{.}21$

Table 4.

	SEASONS				
PARAMETERS	SPRING	SUMMER	AUTUMN	WINTER	MEAN
Δ_1	$-0\overset{''}{.}12$	$-0\overset{''}{.}15$	$-0\overset{''}{.}17$	$-0\overset{''}{.}21$	$-0\overset{''}{.}16$
Δ_2	$0\overset{''}{.}00$	$0\overset{''}{.}00$	$-0\overset{''}{.}06$	$-0\overset{''}{.}02$	$-0\overset{''}{.}02$
Δ_3	$+0\overset{''}{.}04$	$+0\overset{''}{.}03$	$-0\overset{''}{.}16$	$+0\overset{''}{.}02$	$-0\overset{''}{.}02$

Concerning the reality of values $\Delta R''$ it is impposible to give a reliable answer. According to Table 3 they are $+0''.08$, $0''.22$ and $-0''.22$ respectively for the combinations spring - summer, spring - autumn and summer - winter. If one takes into consideration values $\Delta R = +0''.14$, $\Delta R' = -0''.02$ and $\Delta R''$ from the mentioned seasonal combinations, then the seasonal parameters, which ought to be used while treating BCAD instead ΔR, amount for $+0''.08$, $-0''.06$ and $+0''.38$ respectively for mentioned order of combinations. As it can be seen, the correction which had to be applied for combination summer - winter especially is significant. This shows a possible existance of certain influence in the winter period which affects significant systematic errors in refraction determination. Additional examinations are needed to find out about existance and character of these effects.

Concerning the reality of Δb_1 and Δb_2 it can be considered that they are, more or less, real. A satisfactory agreement between these values for different seasons proves this statement. Systematic errors $\Delta b'$ calculated from Δb_1 and Δb_2 are $+0''.11$, $-0''.10$, $+0''.78$ and $-0''.64$ respectively for spring, summer, autumn and winter. Mean seasonal values $\Delta b''$, which represent systematic errors b can be obtained from values $\Delta b = 0''.14$ and $\Delta b'$ for the seasons. These values are $+0''.25$, $+0''.04$, $+0''.92$ and $-0''.50$ respectively for spring, summer, autumn and winter. As it can be seen the most accurate are applied values b from the expresion (2) for summer and significantly inaccurate are the values for the autumn season.

During the observation period seasonal variations of temperature and pressure existed. These parameters have been used for the determination of refraction and for values Δ_1, Δ_2 and Δ_3 from expresion (2) for flexure determination. Besides the normal seasonal temperature variations, a very significant pressure variation existed also, so the mean pressure in autumn and winter period was higher for 4 - 5 mbar than in spring and summer. Such a seasonal variation of the pressure is not the result of real annual variation; it comes from our selection. That was proved from the aerological data obtained from Aerological Observatory.

In Table 4 the mean seasonal values of parameters Δ_1, Δ_2 and Δ_3 are given as well as their complete mean values.

The seasonal variations of the parameters from Table 4 are small except the autumn value Δ_3.

Systematic errors in determination of the temperature, pressure and parameters Δ_1, Δ_2 and Δ_3 could in certain amount affect the obtained seasonal variations, but the reason for existance of significant variations in autumn period for flexure and in winter for refraction and flexure should certainly be looked for on the other side.

4. CONCLUSION

The given analysis showed the existance of significant systematic errors of the values φ, ΔR and b which have been applied during the treatment of BCAD. Significant seasonal variations of ΔR and b exist and they are the source for errors in determination of declination type $\Delta\delta_\alpha$.

The systematic error $\Delta\varphi$ is the consequence of inaccuracy in determination of φ by classical method. This error, because of the existance of accidental errors ε_1 and ε_2 in calculations of δ, was not eliminated even at equal number of observations (n_1 and n_2) in upper and lower culmination, and had a significant role in the formation of errors of type $\Delta\delta_\delta$.

Systematic errors ΔR have primary seasonal character and are specially significant in the winter period. These errors generally affected errors $\Delta\delta_\alpha$.

Systematic errors Δb and $\Delta b'$ showed that at BVC there exists also a so - called, uneliminated flexure, which is very significant at all instruments of this type, except at Pulkovo Vertical Circle. Together with the error $\Delta b = +0''.14$ which affected errors $\Delta\delta_\delta$, significant variations of flexure in autumn and winter periods exist and they caused errors type $\Delta\delta_\alpha$.

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REFERENCES

Mijatov, M., Trajkovska, V.: 1989, *Bull. Obs. Astron. Belgrade*, **140**, 37.

Mijatov, M., Trajkovska, V., Bozhichkovich, D., Sekulović, V.: (Submited to Astronomy and Astrophysics).

Teleki, G.:1967, *Publ. Astron. Obs. Belgrade*, **13**, 1.

Zverev, M. S.: 1946, *Astron. Zh.* **XXIII**, 2, 97.

СИСТЕМАТСКЕ ГРЕШКЕ НЕКИХ ПАРАМЕТАРА СА КОЈИМА ЈЕ ВРШЕНА ОБРАДА БЕОГРАДСКОГ КАТАЛОГА АПСОЛУТНИХ ДЕКЛИНАЦИЈА

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Претходно саопштење

У овом раду су дате систематске грешке примењених вредности географске ширине, поправке константе рефракције и савијања са којима је вршена обрада Београдског каталога апсолутних деклинација (BCAD). Ове систематске грешке су одређене упоређивањем BCAD са фундаменталним каталогом FK5. Констатовано је да средње систематске грешке износе $+0''.14$, $-0''.02$ и $+0''.14$ респективно за наведене параметре и да постоје знатне сезонске промене поправке константе рефракције у зимском и савијања у јесењем и зимском периоду.