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ON THE PHOTOGRAPHIC MAGNITUDE GAIN OF POINT-LIKE SOURCES*

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SUMMARY: starting with the equations for the image illuminance at the focus of the objective, the exposition and sensitivity of the photomaterial, the formula for the weakest reachest magnitude:

$$m = 2.5\log[\tau_0(\frac{D}{F})^2 S_{0.1} t^p (\frac{LF}{C_O C_L})^2] - 15.44$$

and the formula for the limiting magnitude of a point like source:

$$m_L = M + 2.5\log[\frac{S_{0.1}}{S_{0.85}}(\frac{LF}{C_O C_L})^2] - 25.07$$

are derived. The influence of the atmospheric conditions is separated from the influence of the objective and the film. One to the astrophysical significance of the problem, stellar magnitude as photometrical unit is used. It follows that for point like sources the reached magnitude can be increased much more than usually stated (Baum,1962,Shcheglov,1980), especially by taking the camera out of the Earth's atmosphere.

1. INTRODUCTION

Currently the formulae for the limiting photographic magnitude of a point like source are estimated from a quantum approach (Baum, 1955, 1962). However, the reached magnitude can also be obtained from elementary considerations (Charlier, 1889; Scheiner, 1889; Schaberle, 1898; Mineur, 1934). Therefore the classical approach begining with elementary equations of photographic sensitometry will be here modifed by introducing new parameters.

2. PARAMETERS OF THE CAMERA AND FILM ILLUMINATION

As it is well known, if the optical medium is the same on both sides of the objective, the surface brightnes of the image, B1, and that of the source, B, will differ only slightly: $B_1 \doteq \tau_0 B$. Here τ_o is the transmittance of the optical system. The illuminance at the focus is larger that the illuminance in front of the objective by a factor:

$$\tau_0(\frac{D}{d})^2 = \tau_0(\frac{D}{F})^2(\frac{\pi}{4\Omega_0})$$

*This paper is dedicated to Professor Radovan Daničić (1893 - 1979).

Here d is the image diameter at the focus, D and F are the diameter and the focal lenght of the objective, $\pi = 3.14159...$ and Ω_0 is the solid angle of the source seen without the objective. For source with an angular radius $\Delta('')$:

$$\Omega_0 = \pi(\frac{\Delta}{206265})^2 \qquad (1)$$

So for the illuminance at the telescope focus we have:

$$E = \tau_0 \frac{\pi}{4\Omega_0}(\frac{D}{F})^2 E_0 \qquad (2)$$

Where $E_0(1x)$ in astrophisical practice can be expressed using the "brightness" given in stellar magnitudes, m.

$$E_0 = 10^{-0.4m-5.67} \qquad (3)$$

Formula (3) also contains the evaluated influence of the atmospheric absortion in the direction of the zenith. At the zenith distance z the absortion can also be evaluated (Bemporad, 1921, Allen, 1977).

3. RESOLUTION OF THE OBJECTIVE-FILM SYSTEM

The resolution of the film is commonly given as $N(mm^{-1})$, the number of lines per milimeter that can be seen separately. When the receptor is a photoemulsion, the resolution of the objective δ_L is equal to the Airy disc radius (Airy,1835). The resolution of the objective can also be expressed in mm^{-1} with the formula:

$$L = \frac{206265}{F\delta_L} \qquad (4)$$

where the dimensions are F(mm), $\delta_L('')$. Let us introduce a parameter $k(\lambda)$ as: $k = \frac{10^6}{1.22\lambda}$, where λ is the wave lenght of the used light, given in nanometers (nm). Now instead of (4) we can write (like as into: Zakaznov, 1981):

$$L = k\frac{D}{F}$$

so that the resolution of the objective is given using the geometric aperture and the wave lenght of incident light. It is well known, the resolution of the objective - film system can be approximately defined by the formula (for example: Kulagin, 1976):

$$\frac{1}{R} = \frac{1}{L} + \frac{1}{N}$$

It is convenient to introduce a numerical parametar (Tomić, 1983):

$$C_L = 1 + \frac{L}{N}$$

which can have values greater or equal to one. The resolution of the system (in mm^{-1}) is now given with resolution of the objective, L, and C_L as a factor of destruction by the film:

$$R = \frac{L}{C_L}. \qquad (5)$$

It is useful to introduce the angular separation of the film, $\delta_N('')$, for the given objective:

$$\delta_N = \frac{206265}{FN}$$

where the units of N and F are as before. The resolution of the camera as the system made from objective and film is now given as:

$$\delta_s = \delta_L C_L \qquad (6)$$

Further, the ratio between angular radius of source, Δ, and δ_s: $C_0 = \frac{\Delta}{\delta_s}$, can be the criterion for the regime of work. The photography of stars is a process with duration into the time. So due to the atmospheric scintillation, recorded stellar angle can show spread. The parameter C_0, "relative spreading" for point like sources is determined with:

$$C_0 C_L = \frac{\Delta}{\delta_L} \geq 1$$

If $\Delta \leq \delta_L$, we have $C_0C_L = 1$,and for $\Delta > \delta_L$ it is $C_0C_L > 1$.

In this way, an important distinction is made between the camera parameters, which practically remain unchanged for given objective and film, and the observational parameters, that can vary considerably.

4. ILLUMINANCE BY POINT LIKE SOURCES

When photographing stars the illuminance at the focus is given with (2), i.e. for point like sources the important characteristic is the illuminance, not often assumed brightness (Hartman, 1898, Gray, 1976). When $\Delta > \delta_s$ it is necessary for point like sources to insert Eq. (1) and (3) into Eq. (2).When $\Delta \leq \delta_s$ instead of formula (6) and the expression for the Airy radius,one can use:

$$\delta_L = \frac{\lambda}{3.973D} \simeq \frac{\lambda}{4D} = \frac{206265}{kD}$$

with the units $\delta_L(''), \lambda$ (nm), D(mm). Formula (2) has the form:

$$E = \frac{\pi}{4}\tau_0(\frac{D}{F})^2(\frac{kD}{C_L})^2 10^{-0.4m-5.67}$$

Thus we have obtained the illuminance by a point like source of magnitude m, at the focus of a teleskope with the parameters τ_0, D, F, if the radius of spreading of the image caused by atmospheric disturbances is smaller than δ_s.

5. THE REACHED STELLAR MAGNITUDE

The stellar magnitude reached during the time t(s) is given, as it is well known, by (Altman, 1977):

$$Et^p = \frac{0.8}{S_{0,1}}$$

in which $S_{0,1}$ is the photomaterial sensitivity for the level of blackening 0,1 above the fog level, given in ASA units, and p is the Schwarzschild's exponent. Depending on the observational conditions, there are two possible regimes of work, $\Delta > \delta_s$ or $\Delta \leq \delta_s$, by which a different stellar magnitude can be reached:

$$m_{1,2} = 2.5\log[(\frac{D}{F})^2 S_{0,1}\tau_0 t^p] + \mu_{1,2} \qquad (7)$$

$$\mu_1 = 11.13 - 5\log\Delta, \qquad \Delta > \delta_s \qquad (7.1)$$

$$\mu_2 = 2.5\log(\frac{kD}{C_L})^2 - 15.44, \qquad \Delta \leq \delta_s \qquad (7.2)$$

In practice, enough large objectives and emulsions of good quality, the case $\Delta > \delta_s$ is more frequent. When $\Delta \leq \delta_s$ formulae (7) and (5) gives:

$$m_2 = 2.5\log(\tau_0 D^2) + 2.5\log(R^2 S_{0,1}) +$$

$$2.5\log t^p - 15.44$$

which justifies idea (Paetzold, 1953) that the real sensitivity of the photomaterial is determined by $R^2 S$. The equations (7) ÷ (7.2) can be put in one:

$$m = 2.5\log[\tau_0(\frac{D}{F})^2 S_{0,1} t^p (\frac{LF}{C_0 C_L})^2] - 15.44 \qquad (8)$$

Here m is the stellar magnitude reached by using a camera with the parameters τ_0, D, F and film with parameters $S_{0,1}$, N when the spreading due to atmospheric disturbances is Δ, and the time of illumination t(s).

By applying formula (8), one obtains values in good agreement with the measured data. For example, the 508 cm Hale telescope in $t = 30min$ reaches $m = 23.6$ (Shcheglov, 1980). On the other hand, our formula (8) with $D = 5000$ mm, $F = 18550$ mm,$N = 400$ mm^{-1}, $2\Delta'' = 1.3''$,$S = 125$ ASA, $p = 1$ gives $m = 23.4$.

6. THE LIMITING PHOTOGRAPHIC STELLAR MAGNITUDE

The time of illumination T^p, after which the sky leaves a trace whose density of blackening is equal to the optimal value, is the limiting time of illumination. It is given as:

$$T^p = \frac{10}{E_1 S_{0,85}} \qquad (9)$$

where $S_{0,85}$ is the sensitivity of the photomaterial at the level of 0,85 above the fog, and:

$$E_1 = \frac{\pi}{4}\tau_0(\frac{D}{F})^2 10^{-0.4M+4.96} \qquad (10)$$

is the illuminance from the sky at the objective focus. The brightness of the sky M is given in $\frac{m}{('')^2}$ units, that are linked with the brightness $B(\frac{cd}{m^2})$ in the equation (see e. q. Tomić, 1981):

$$M = 12.39 - 2.5\log B.$$

From Eq. (7), (9) and (10) two formulae for the limiting stellar magnitude are obtained:

$$m_{1,lim} = M - 5\log\Delta + 2.5\log(\frac{S_{0,1}}{S_{0,85}}) + 1.50 \qquad (11)$$

$$m_{2,lim} = M + 5\log(\frac{kD}{C_L}) + 2.5\log(\frac{S_{0,1}}{S_{0,85}}) - 25.07 \qquad (12.1)$$

The last equation can be brought to a form:

$$m_{2,lim} = M + 5\log F - 2.5\log(\frac{S_{0,85}}{S_{0,1}}) +$$

$$+5\log(\frac{L}{C_L}) - 25.07 \qquad (12.2)$$

This form is similar to one obtained from the quantum approach, which indicates the qualitative agreement of both approaches. According to Baum's theory, for regime of saturation:

$$m_L = M + 2.5\log F - 2.5\log(1+R_0) - 2.5\log 2\Delta -$$

$$-2.5\log b + 1.25\log G_0$$

where b, R_0, G_0 caracterize quantum quantities. (See Baum, 1962). But, according to (5), equation (12.2) can take another form, which is also of interest:

Fig. 1

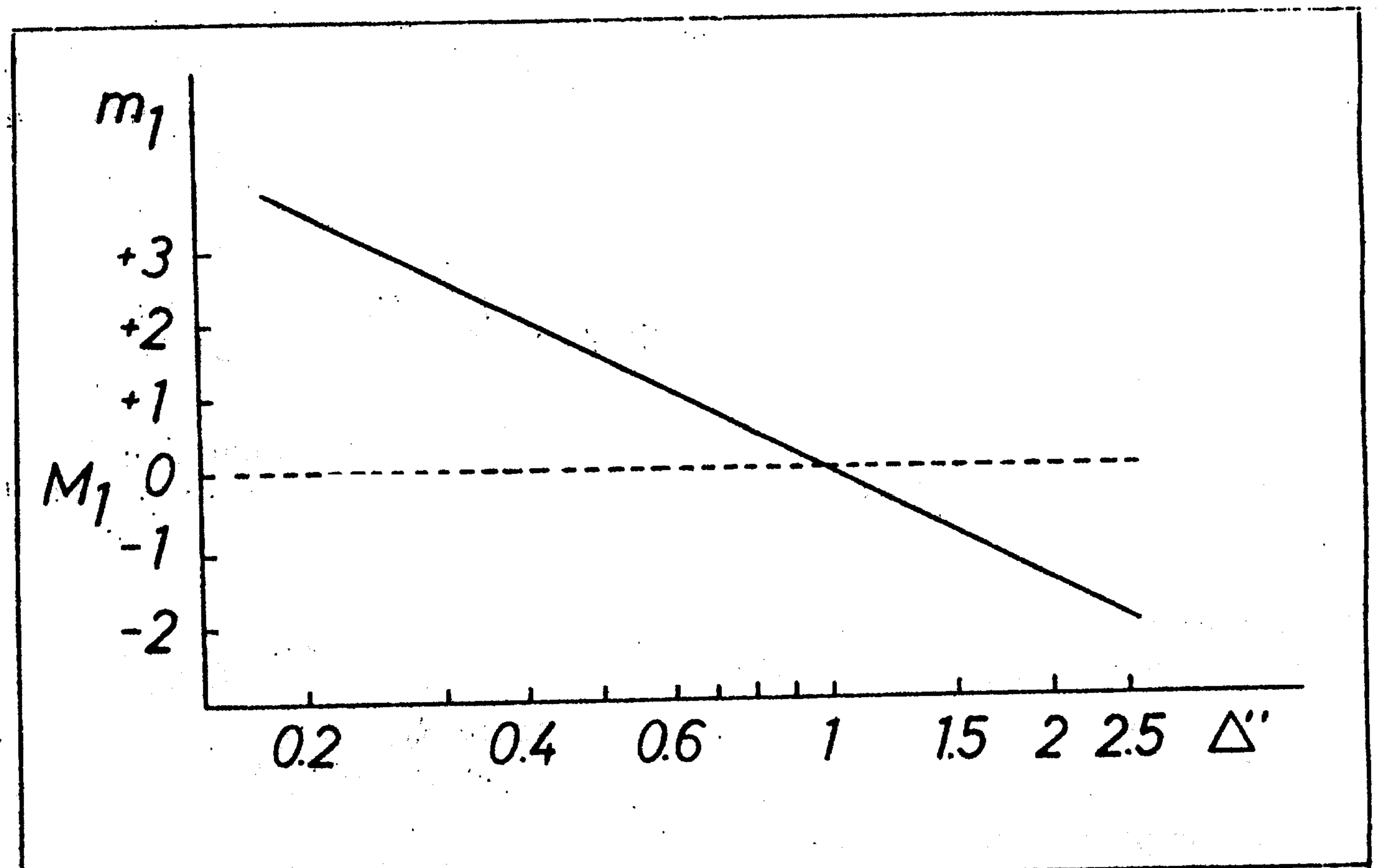

Fig. 2

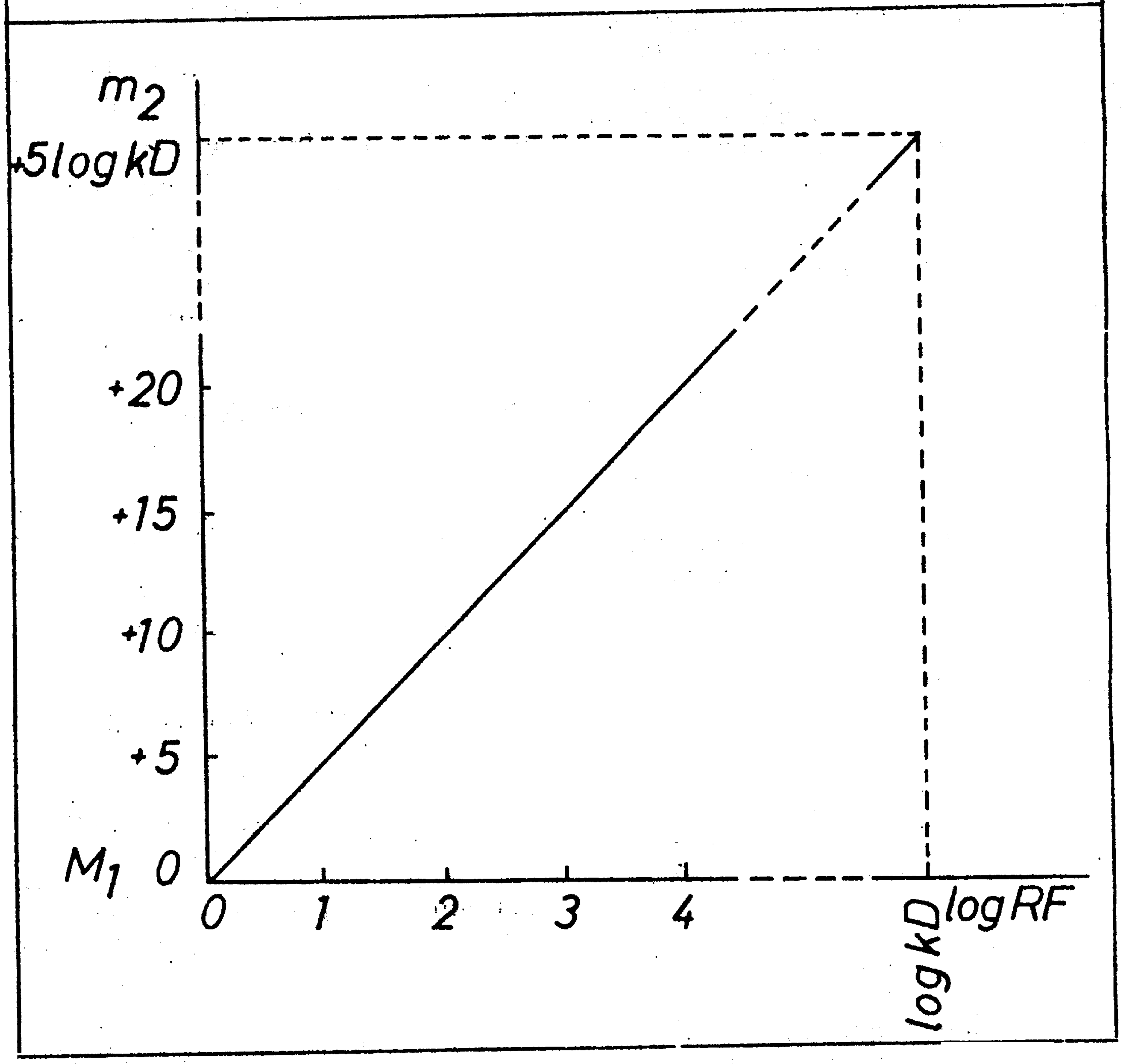

Fig. 1. The limiting apparent stellar magnitude m_1 as a function of the atmospheric spreading Δ, according to Eq. (11).

Fig. 2. The limiting apparent stellar magnitude m_2 as a function of the system resolution - focal length product (RF), calculated according to Eq. (12.1).

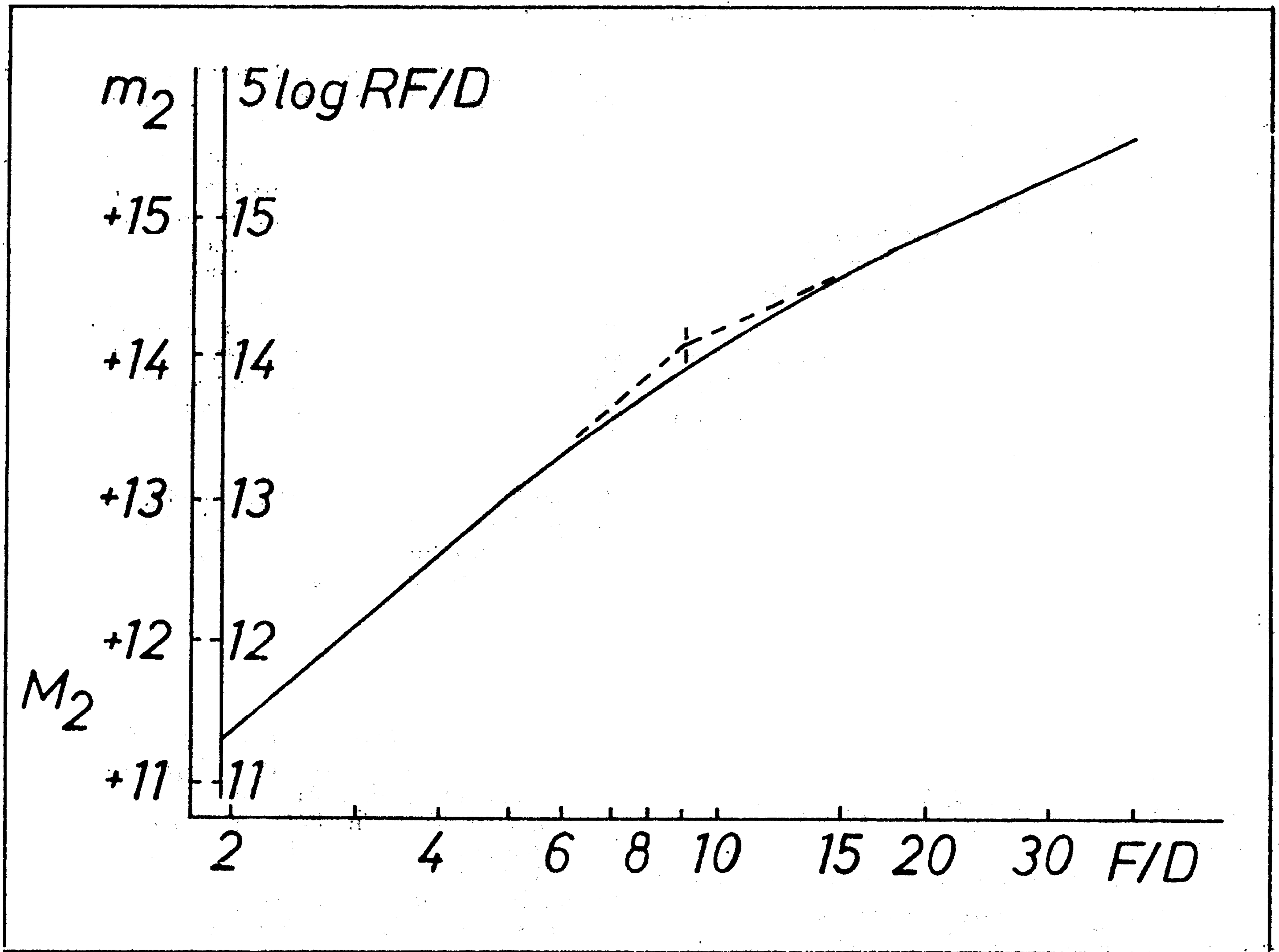

Fig. 3. The case $\Delta \leq \delta_s$: The limiting magnitude m_2 as a function of the focal ratio, calculated according to Eq. (12.3) for $N = 100, D = 100, k = 1800$. Here: $M_2 = M_1 + 5\log D(mm)$.

$$m_{2,lim} = M + 5\log D + 5\log(R\frac{F}{D}) +$$

$$+2.5\log(\frac{S_{0,1}}{S_{0,85}}) - 25.07 \qquad (12.3)$$

Now, m is expressed as a function of aperture D, and relative aperture $\frac{F}{D}$, conected with resolution R. From Eq. (8) to (10) the stellar magnitude is in the general case obtained as:

$$m_{lim} = M + 2,5\log[(\frac{LF}{C_0 C_L})^2 \frac{S_{0,1}}{S_{0,85}}] - 25.07 \qquad (13)$$

which is defined by the sky brightness M, objective parameters F,L. Here, included are the film parametres $S_{0,1}, S_{0,85}$, N and the atmospheric disturbance - spreading of the image C_0.

The values obtained by application of (13) are in excellent agreement with the observational results. For example (Kopilov et all, 1979), the Russian 6 m teleskope is characterized by $D = 6000\ mm, F = 24000\ mm$, which for $N = 400\ mm^{-1}$, $2\Delta = 1.3''$, $\frac{S_{0,1}}{S_{0,85}} = 0.8$ gives: $m_{lim} = M + 2.9$, and finally: $m_{lim} = 24$. A similar value is obtained for the ESO 3,6 m teleskope (Laustsen, 1977, Shcheglov, 1980).

7. DISCUSSION

The case when $\Delta > \delta_s$ Eq. (11.1) During the time T^p a stellar magnitude weaker than the sky brightness can be reached if $\Delta < 1''$. For the developing conditions commonly used in astronomical practice (e. q. Brejdo, 1973) the ratio $\frac{S_{0,1}}{S_{0,85}}$ is not less than 0,5 so that it is not difficult to obtain images of stars fainter than the brightness of the sky. (Figure 1.) In this figure the symbol M_1 denotes the quantity: $M_1 = M + 2.5\log(\frac{S_{0,1}}{S_{0,85}}) + 1.50$. The extreme magnitude range of the teleskope does not depend on the objective parameters, but on the visibility conditions and only partly on the essential property of the photomaterial, the number of grains per area unit is taken into account, but the criterion $\Delta > \delta_s$ has excluded its influence, as it has excluded the influence of the geometrical parameters of the objective. (Formulae (4), (5), (6).)

The case when $\Delta \leq \delta_s$ Equation (12.2) needs no comment. However, it should be emphasized that $\frac{L}{C_L}$ also contains the influence of the objective radius and geometrical aperture which can be seen in (12.3). Since in astronomy $\frac{F}{D} > 1$, it follows that $5\log[\frac{RF}{D}] > 0$. The influence of the focal length, which is dominant according to (12.2) and (13) appears with the same sign exactly in this term, but at the same time it is related with the diameter of the objective, and information capacity of the photomaterial. (Figure 2.) From it follows the comparison of equations (12.2) and (11), that RF and $\frac{1}{\Delta}$ are physically equivalent quantities. (See also Figures 1 and 2.) It is indicated by the fact that after large values of F only the contrast against the fog becomes important (Shcheglov, 1980, Brejdo, 1973 etc.), and it is reciprocal to the ratio $\frac{D}{F}$.

In Figure 3 we see that for $D = const$ the function $5\log(\frac{RF}{D}) = f[\log(\frac{F}{D})]$ has two nearly linear parts, that intersect for $\frac{F}{D} \simeq 9$. That means that the optimal aperture for a point like source is exactly $\frac{F}{D} = 9$. The same result is obtained by the quantum approach (e. q. Shcheglov, 1980) Othervise, the graph: $m_{2,lim} = f(\log F)$ has an almost likely shape as in Figure 2.

8. CONCLUSIONS

Independently of the equations for photographic gain of point like sources containing quantum parameters, equivalent equation with parameters of classical photographic sensitometry can be derived. Both cases, $\Delta > \delta_s$ and $\Delta \leq \delta_s$, are integrated into same equation (8) for reached and equation (13) for limiting stellar magnitude. Agreement of results for this two approaches is satisfactory.

The photographic gain for point like sources for classical approach is defined by objective diameter D, aperture $\frac{D}{F}$ and information capacity (resolution) of the film or, what is equal, the focal length F, objective and film resolutions L and N, while it practicaly does not depend on the photomaterial sensitivity. It should be emphasized that this is the case **only when the extreme gain is in question.** In an actual case when the illumination time is shorter than the limiting time of illumination, the photomaterial sensitivity and time of illumination appear. (Eq. 8.)

In absence of the Earth's atmosphere (for example, an orbital or Lunar observatory) the range of observed magnitudes of point like sources could be **considerably enlarged**, since the condition $\Delta \leq \delta_s$ can be fulfilled. As angular dimensions of object appear here with multiplyer 5, and in Baum's quantum approach with multiplyer 2,5 our equation predict a greater magnitude gain. In that case the limiting factors will be exposure time and accuracy of stabilization of the camera in the direction of the observed objects.

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О ДОСТИГНУТОЈ ФОТОГРАФСКОЈ МАГНИТУДИ ТАЧКАСТИХ ИЗВОРА

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Полазећи од једначина за осветљеност лика у жижи објектива, експозицију и осетљивост фотоматеријала, изведене су формуле за најслабију достигнуту и за граничну величину интензитета тачкастог извора. Утицај атмосферских услова је издвојен од утицаја објектива и филма. Због значаја третираног проблема за астрофизику као фотометријска јединица коришћена је звездана величина. Добијено је да за тачкасте изворе достигнута звездана величина ван Земљине атмосфере може порасти више него што се обично сматра.