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# IS THE CHAOTIC CLOCK TICKING CORRECTLY?

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**SUMMARY: We performed an extensive analysis of the dynamics of all presently known members of the Veritas asteroid family, in order to check the estimate of its age given originally by Milani and Farinella (1994), who used the so-called "chaotic chronology" method for this purpose. We started from a much larger sample of family members and carried out many more numerical integrations of the entire family and, in particular, of the chaotic bodies; integrations of these latter embraced typically 200 Myr per body, covering a total of about 2.1 Gyr. We show that the dynamics in the region of the phase space occupied by the family is more complex than previously believed, and that there are bodies with motions ranging from remarkable stability to strong chaos, due primarily to the effects of the 21/10 Jovian mean motion resonance and its harmonics involving slow angular variables. Seven strongly chaotic bodies exhibit a qualitatively similar behavior, in agreement with the predictions of Milani et al. (1996), but also a number of previously unknown features, the most important one pertaining to the fact that some of the chaotic asteroids, including the largest member of the family 490 Veritas itself, can be captured into the resonance and stay there for a very long time in a quasi–stable state. Although, in general, our results do not contradict the conclusions of Milani and Farinella and fit reasonably well with their estimate of the family age, there are also some interesting results of this analysis, which open new questions and require a more thorough and dedicated investigation.**

## 1. INTRODUCTION

The ages of the asteroid families represent one of the most important pieces of information still missing and badly needed in order to calibrate and constrain the current models of the collisional evolution of the asteroid belt, and of the Solar System as a whole. At the same time, this is one of the most difficult problems to deal with in asteroid research, as no "direct" evidence is available yet (probably until a sample–return mission to a family member will have been carried out). Several attempts have been made so far in this direction (c.f. Farinella et al. 1989, Farinella 1994), but with no real success. Recently, however, Milani and Farinella (1994) found a way to "indirectly" estimate at least an upper bound to the age of a family (more precisely, of a particular family – that of 490 Veritas), by means of the so–called *chaotic chronology* method, based on purely dynamical arguments. This result has attracted a great deal of attention, and, as a consequence, a number of papers have appeared in the past few years devoted just to the study of this family.

The Veritas family is a small, compact one, located in the outer part of the main belt, in a region interesting and important from both the dynamical and the physical points of view. The reliability of its membership is rated as very high, and it is also considered to be well defined in the proper element phase space (Zappalà et al. 1995). The dynamical peculiarity of the Veritas family that has been recognized and used by Milani and Farinella (1994) pertains to the fact that the proper elements of a couple of family members (including the largest one, 490 Veritas itself) chaotically wander in the phase space of proper elements, instead of remaining nearly constant in time. By integrating the orbits of the seven numbered family members (plus four background objects) for 72 Myr in the past, they demonstrated that the rate of chaotic diffusion undergone by the two member asteroids might be large enough that they exit from the region occupied by the family in the proper element space, after a characteristic time much shorter than the lifetime of the Solar System (some 50 Myr). As the physical characteristics of asteroid 490 Veritas itself are very similar to those of the rest of the family, thus undoubtedly establishing it as family member (unfortunately, no data are available for the other wandering object 3542 $1964TN_2$, although, according to Migliorini et al. 1995, it is highly unlikely that any of the proposed family members can be a chance interloper), the characteristic time found from the dynamical diffusion effect should provide an upper bound estimate for the age of this family.

A detailed analysis of the dynamics of the two Veritas family asteroids having chaotic orbits has been performed by Milani et al. (1996), in the framework of a study of the phenomenon of *stable chaos* in the asteroid belt. The Veritas family members are found to be a paradigm for this particular type of chaotic motion, and therefore especially suitable for its investigation, as no other causes of chaotic behavior are relevant in the region occupied by the family. Milani et al. found that chaos in this case is due to high order mean motion resonances with Jupiter, in combination with secular perturbations on the perihelia and the nodes of the asteroids. These perturbations push a number of critical arguments in and out of the resonance and shift the orbit from one resonance to another, in a typical irregular manner. The proper elements are therefore less stable and a slow diffusion is observed in eccentricity and inclination, which may eventually bring chaotic bodies out of the family.

Finally, Di Martino et al. (1997) reported the first optical reflectance spectra for seven members of the Veritas asteroid family and another body which joins the family when a slightly more relaxed criterion for membership is adopted. They found a slope variation in these spectra, spanning a range that includes slopes typical of all the low–albedo, primitive bodies (from C to D type). The possible explanations of this slope gradient, according to the authors, are two: it is due either to a space weathering process similar to those affecting S–type asteroids, or to an alteration of the parent body's interior due to extended thermal episodes prior to the family–forming breakup event. In addition, they reconstructed the original velocity field of the family, deriving it under the tentative assumption that the present position of the largest body 490 Veritas is representative of its true position at the instant of breakup. Even if this latter assumption may be unrealistic (due to the chaotic motion of Veritas), they found that the velocity field can be well represented by a "jetlike" planar structure, which, in a qualitative sense, does not change even when the position of the barycenter of the family is moved.

In view of the above results regarding the Veritas family, and of the current understanding of chaotic phenomena in the motion of asteroids in general, the rationale for the present paper can be summarized as follows:

- the conclusions of Milani and Farinella (1994), though quite sound and convincing in the framework of the then available data, were drawn on the basis of a rather limited set of long–term integrations, and a less complete knowledge of the membership of the family, i.e. of its size in the space of proper elements (which, in turn, might significantly affect the estimate of the family age). With a computing power enhanced dramatically in the last few years, we are now able to perform many more experiments, covering much longer time spans than Milani and Farinella originally did, thus ensuring that any conclusion based on the statistics of chaotic orbits will become more reliable; moreover, the membership of the family is meanwhile increased so that it currently consists of 11 numbered and 11 multi–opposition and single–opposition bodies with reasonably well determined orbits (i.e., with a quality code for the osculating orbital elements $QCO < 20$; see Milani et al. 1994);
- an improved understanding of the dynamics involved with the chaotic Veritas family members may provide a capability of predicting the stable chaos type behavior for bodies in the same narrow semimajor axis region, and a number of newly recognized members of the family are found to be located just there. Having more bodies which exhibit a similar chaotic behavior would give a more comprehensive insight in the complex dynamics involved, possibly strengthen and/or refine the existing results and conclusions, and even perhaps offer some entirely new explanations and ideas.

## 2. PROCEDURES AND RESULTS

In order to produce comparable results, we have in general closely followed the procedures of numerical integration and output analysis used by Milani and Farinella (1994; hereinafter referred to as M&F). Thus, we have performed numerical integrations by means of the same software (ORBIT9a, kindly provided by A. Milani), have analyzed the output by using the same routines and procedures (GIFFV software, again provided by A. Milani), and have used the same software to compute proper elements (Milani and Knežević 1994). Only in minor details, which could not affect the comparisons, we have changed the original approach, and these changes will always be mentioned and briefly explained, if necessary, in the following.

**Table I.** Results from the 10 Myr integrations. The columns contain the asteroid identification, the mean ($\bar{a}$) and RMS ($\sigma_a$) values of the filtered semimajor axis over the integration time span, the Lyapunov characteristic exponents LCE and Lyapunov times ($T_L$). Only the objects with shortest Lyapunov times entered the second phase of our analysis.

| Asteroid | $\bar{a}\,[AU]$ | $\sigma_a\,[AU]$ | $LCE\,[yr^{-1}]$ | $T_L\,[yr]$ |
|---|---|---|---|---|
| 490 Veritas | 3.17397 | 0.00125 | $9.775 \times 10^{-5}$ | 10 250 |
| 844 Leontina | 3.19575 | 0.00008 | $5.494 \times 10^{-7}$ | 1 500 000 |
| 1086 Nata | 3.16536 | 0.00003 | $2.249 \times 10^{-7}$ | $>$ 1 500 000 |
| 2147 Kharadze | 3.17098 | 0.00003 | $7.822 \times 10^{-6}$ | 127 800 |
| 2428 Kamenyar | 3.16997 | 0.00004 | $2.662 \times 10^{-6}$ | 375 700 |
| 2934 Aristophanes | 3.16659 | 0.00005 | $4.195 \times 10^{-6}$ | 238 400 |
| 3090 Tjossem | 3.16906 | 0.00004 | $3.971 \times 10^{-7}$ | $>$ 1 500 000 |
| 3542 1964 $TN_2$ | 3.17410 | 0.00122 | $1.115 \times 10^{-4}$ | 8 971 |
| 5592 Oshima | 3.16844 | 0.00005 | $8.776 \times 10^{-8}$ | $>$ 1 500 000 |
| 5594 1991 $NK_1$ | 3.16765 | 0.00032 | $3.387 \times 10^{-5}$ | 29 520 |
| 7231 1985 $TQ_1$ | 3.16580 | 0.00004 | $3.160 \times 10^{-8}$ | $>$ 1 500 000 |
| 1976 $QL_2$ | 3.17038 | 0.00008 | $1.083 \times 10^{-5}$ | 92 340 |
| 1981 $ES_9$ | 3.16474 | 0.00003 | $3.013 \times 10^{-7}$ | $>$ 1 500 000 |
| 1981 $EE_4$ | 3.17405 | 0.00124 | $1.075 \times 10^{-4}$ | 9 302 |
| 1981 $EM_{10}$ | 3.17178 | 0.00006 | $8.808 \times 10^{-6}$ | 113 500 |
| 1981 $ER_{34}$ | 3.16538 | 0.00003 | $2.785 \times 10^{-7}$ | $>$ 1 500 000 |
| 1991 $PW_9$ | 3.16425 | 0.00003 | $3.141 \times 10^{-7}$ | $>$ 1 500 000 |
| 2123 $PL$ | 3.17409 | 0.00125 | $1.122 \times 10^{-4}$ | 8 913 |
| 4107 $PL$ | 3.16364 | 0.00003 | $4.194 \times 10^{-7}$ | $>$ 1 500 000 |
| 4573 $PL$ | 3.17420 | 0.00112 | $1.218 \times 10^{-4}$ | 8 210 |
| 1118 $T3$ | 3.17410 | 0.00119 | $1.048 \times 10^{-4}$ | 9 542 |
| 1122 $T3$ | 3.18258 | 0.00004 | $2.248 \times 10^{-7}$ | $>$ 1 500 000 |

As already mentioned, at present the Veritas family consists of a total of 22 objects. For all of them, we have first performed a preliminary numerical integration, lovering 10 Myr, which served solely for the purpose of establishing their dynamical state; we also computed the mean/proper semimajor axes and their RMS deviations applying an on–line filtering procedure (see Carpino et al. 1987), as well as derived the Lyapunov characteristic exponents and the corresponding Lyapunov times (the inverse of the slope). The initial conditions for these integrations were taken from the asteroid osculating orbital elements data base by E. Bowell (Bowell et al. 1994); the results are summarized in Table I.

Some simple conclusions can be drawn by inspecting the data contained in Table I. It is clear that, from the point of view of the stability of their motion, there are three distinct groups of objects in the family. Stable orbits are characterized by a very small LCE and a Lyapunov time on the order of millions of years (note that $T_L$ values in excess of 1.5 Myr have no meaning in our case, because of the short time span covered by the integrations). A group of objects with $T_L$ on the order of $10^5$ yr can be considered as being in an intermediate state between stability and chaos (they are only marginally chaotic, and no macroscopic instability due to chaos was observed for these objects). Finally, there are 7 strongly chaotic family members, and these are the only objects which we are actually interested in. Their Lyapunov times are on the order of $10^4$ yr, and they also exhibit comparatively large variations (although no long term evolution) of the semimajor axes, thus representing typical examples of stable chaos.

All but one of the chaotic objects have their average proper semimajor axes clustered within a narrow strip, located at about 3.174 AU; the only object that is out of this main chaotic strip (5594), is located at a slightly lower semimajor axis, but it is also somewhat less chaotic. All this is in good agreement with the results of Milani et al. (1996), who found that chaos in this region is due to the high order mean motion resonance 21/10, and its secular harmonics (with critical arguments containing different combinations of perihelia and nodes). As explained in that paper, the region of influence of a high order mean motion resonance is small, but its main effect shows up at some distance from the resonant value of the fast variables (mean anomalies), depending on the particular combination of slow angles involved in the critical argument. Even if the chaotic region is presumably as wide as $\simeq 3 \times 10^{-3}$ AU (judging from the values of $\sigma_a$ for chaotic bodies; see Table I), the averaged semimajor axes of chaotic orbits cluster, as expected provided the time span covered with integration was long enough, near the center of the chaotic region.

A comment is in order here: in view of the limited size of the chaotic region we are interested in, the accuracy of the osculating orbital elements of the bodies in question (especially their semimajor axes) is of critical importance, in particular to decide whether all the bodies we see inside the chaotic strip really belong there, or they just by chance might only appear to be there. Out of the seven chaotic members of the extended Veritas family, only three are numbered asteroids (that is, with their orbits known to an accuracy better than $10^{-4}$ AU in semimajor axis). The other four bodies are all single opposition ones, two of which (1981 $EE_4$ and 2123 $PL$) have orbits that, according to Bowell's 1997 data base, can be rated as reasonably reliable (error in osculating semimajor axis on the order of $10^{-3}$ AU, and anyway presumably smaller than the width of the chaotic region), whereas the remaining two ((1118 $T3$ and 4573 $PL$) have rather poorly determined orbits, with errors exceeding the chaotic region size. The latter bodies should therefore be considered as analogous to "fictitious" bodies, and we applied for them the usual trick of producing "clone orbits", with initial semimajor axes changed by an amount corresponding to the standard deviations of the nominal ones; the clone integrations have then been analyzed along with the nominal ones (unfortunately, with little success).

The next thing we have done was to compute the time series of proper elements for all the integrated bodies, filtering the results again to remove all the remaining perturbations up to 40,000 yr of period. We have then plotted the 10 Myr time evolution of the proper eccentricity and proper sine of inclination for all the 22 members of the Veritas family (see Fig. 1, to be compared with Fig. 1a of M&F), in order to reveal any change in the size and boundaries of the extended family with respect to the previous situation (see also a similar plot in Di Martino et al. 1997, which contains as an additional information the diameters of the family members). Due to the remarkable stability of the proper elements for the majority of family members the general picture did not change very much in a qualitative sense. Quantitatively, however, there are some changes, but they are definitely not so large to affect significantly the overall compactness of the family, or to cause a dramatic increase of its size.

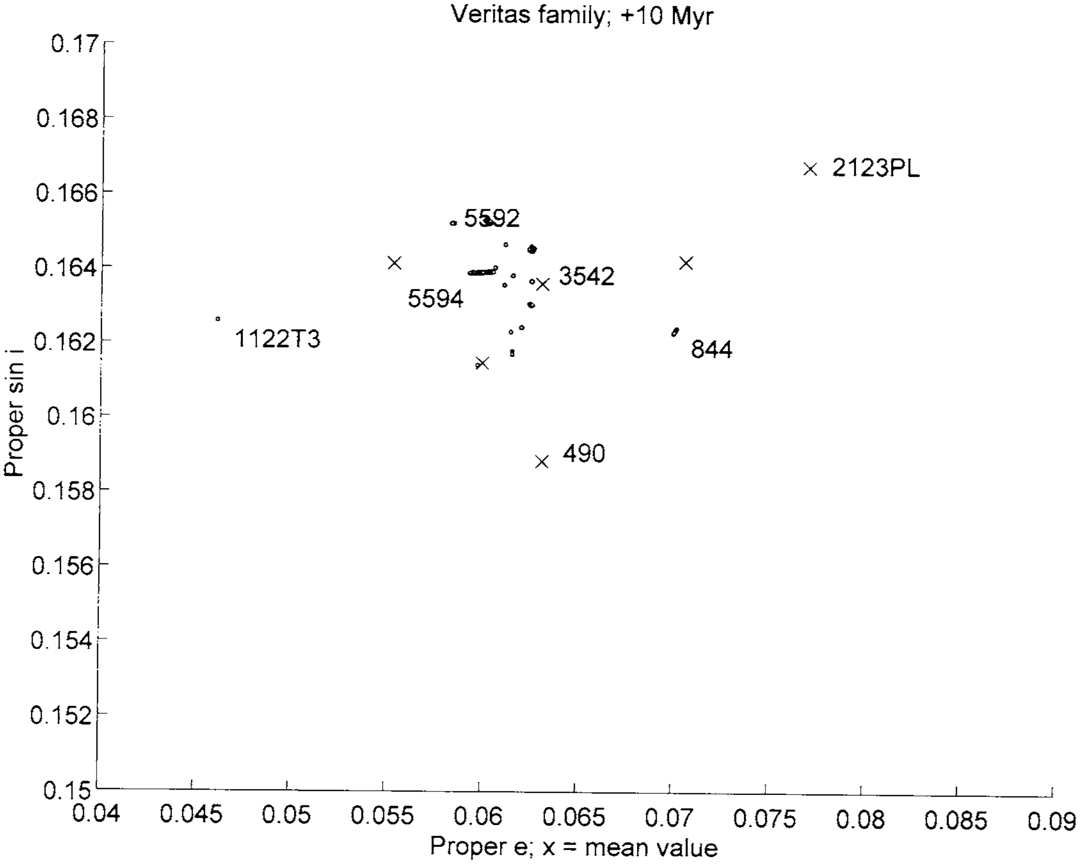

**Fig. 1.** *Time evolution of the proper eccentricity and proper sine of inclination for all the Veritas family members. Due to the remarkable stability of proper elements the entire variation is confined within a dot on the plot for the majority of the bodies; a short line near the center of the family corresponds to 2147 Kharadze, which exhibits a residual long term oscillation of the proper eccentricity. Proper elements are plotted once every 40 000 yr; for the chaotic bodies crosses indicate the corresponding averages over the integration time span.*

Perhaps the most important change with respect to the previous situation is due to the inclusion of asteroid 844 Leontina in the family. This is the second largest body in the family, and it has been included in the family only recently (note that in the M&F paper it was still considered as a background object). It is rather far away from the family in proper semimajor axis, and at the very edge of the family in proper eccentricity (note the relatively large difference between the proper eccentricities found by M&F and those computed in this paper; the latter are in good agreement with the current values provided by Milani and Knežević 1994 —version 6.8.5.— and used by Zappalà et al. 1995 to define families). If, in addition, one takes into account that this body is found to be difficult to fit into the possible collisional model and velocity field of the family, and that its spectrum has the largest slope of all the observed family members (Di Martino et al. 1997), one wonders whether its family membership may be only an artifact of the statistical clustering technique (according to Zappalà et al. 1995, 844 separates from the other family members at a "distance" level not very much below the QRL for the outer–belt region). Anyway, from the point of view of the size of the family, the issue is not so critical, since in the proper $(e, \sin i)$ plane 844 Leontina is not very far from the rest of the family, and there are some other newly added members which appear to be even farther away.

In Fig. 2 a typical example is shown of the 10 Myr evolution of proper elements for one of the chaotic objects (to be compared with Fig. 1b of M&F), giving rise to "clouds" in the phase space. Again one point is plotted every 40,000 yr. We have selected on purpose an object located currently (but also on the average) near the geometrical center of the family, to show that even in that "unfavourable" case chaotic evolution can readily bring the body at the very edge of the family, and probably out of the family in an experiment of longer duration. Similar plots can be shown for all the chaotic members of the family, those close to the family borders already covering an area in the proper $(e, \sin i)$ plane extending well out of the family.

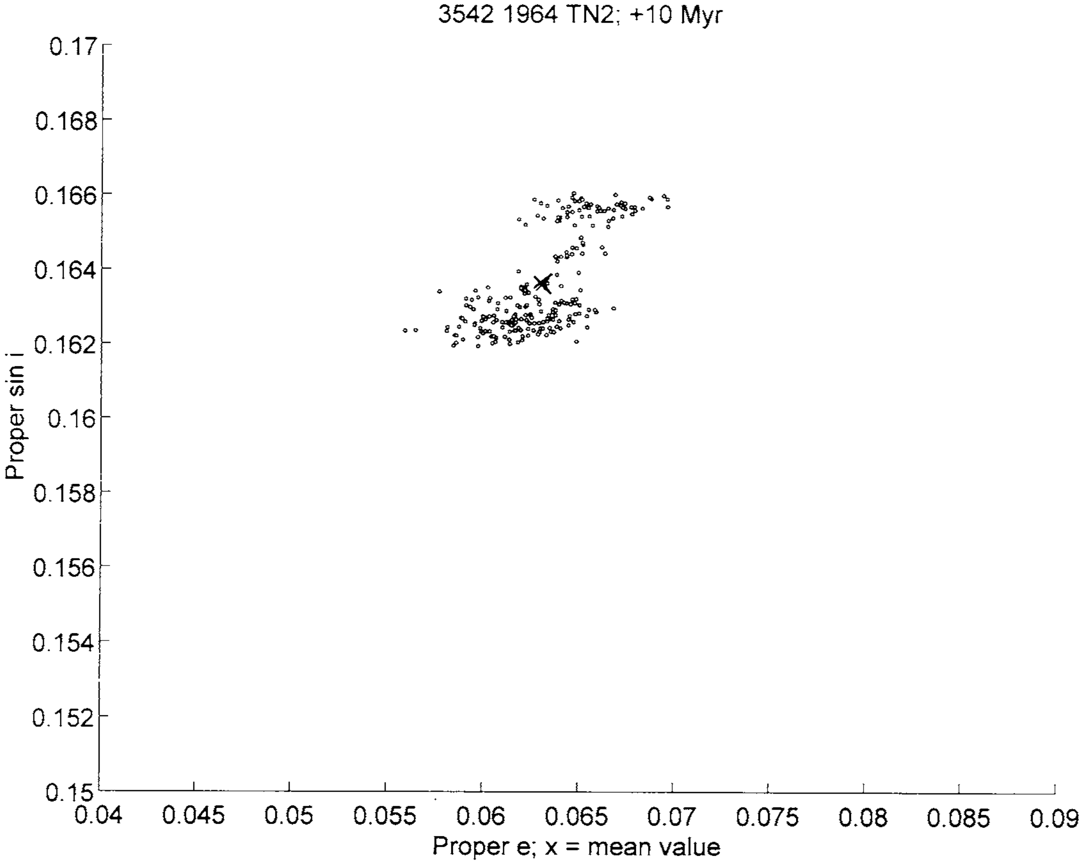

**Fig. 2.** *Time evolution of proper eccentricity and proper sine of inclination for one of the chaotic bodies, namely 3542 1964* $TN_2$. *This asteroid is, on the average, located near the center of the family, but its proper elements change in such a way to bring it close to the border of the family in the relatively short interval of time covered by the integration. It is therefore plausible to expect that a longer integration would show this body to exit the family region. Similar plots can be produced for all the chaotic members of the family.*

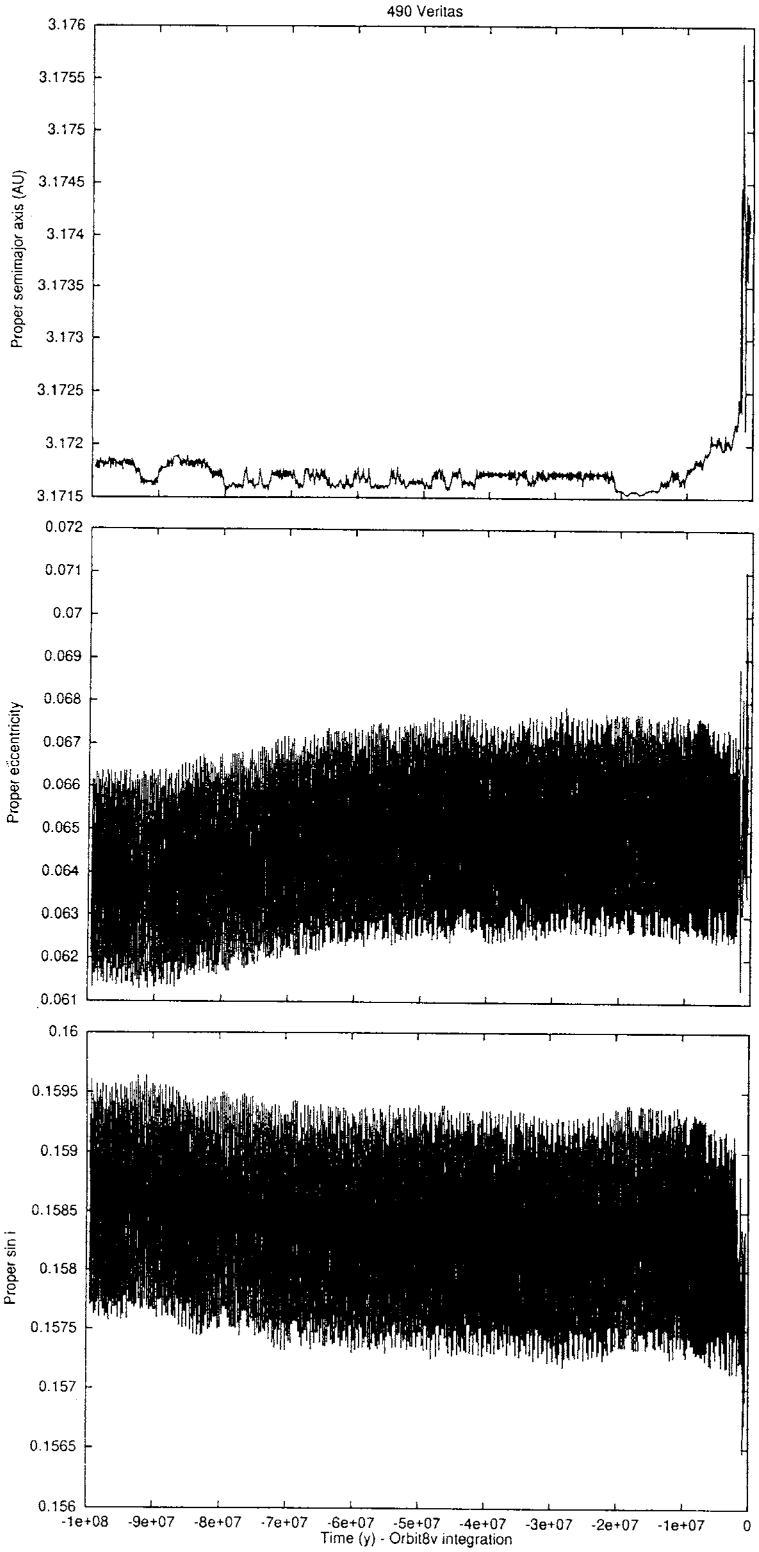

**Fig. 3a.** *Time evolution of the proper semimajor axis, proper eccentricity and proper sine of inclination for 490 Veritas (100 Myr backward integration). A "capture" into a high order mean motion resonance (a harmonic of the 21/10 resonance) occurred and lasted almost for the entire integration time span. This process may affect the interpretation of the results in terms of the age of the family.*

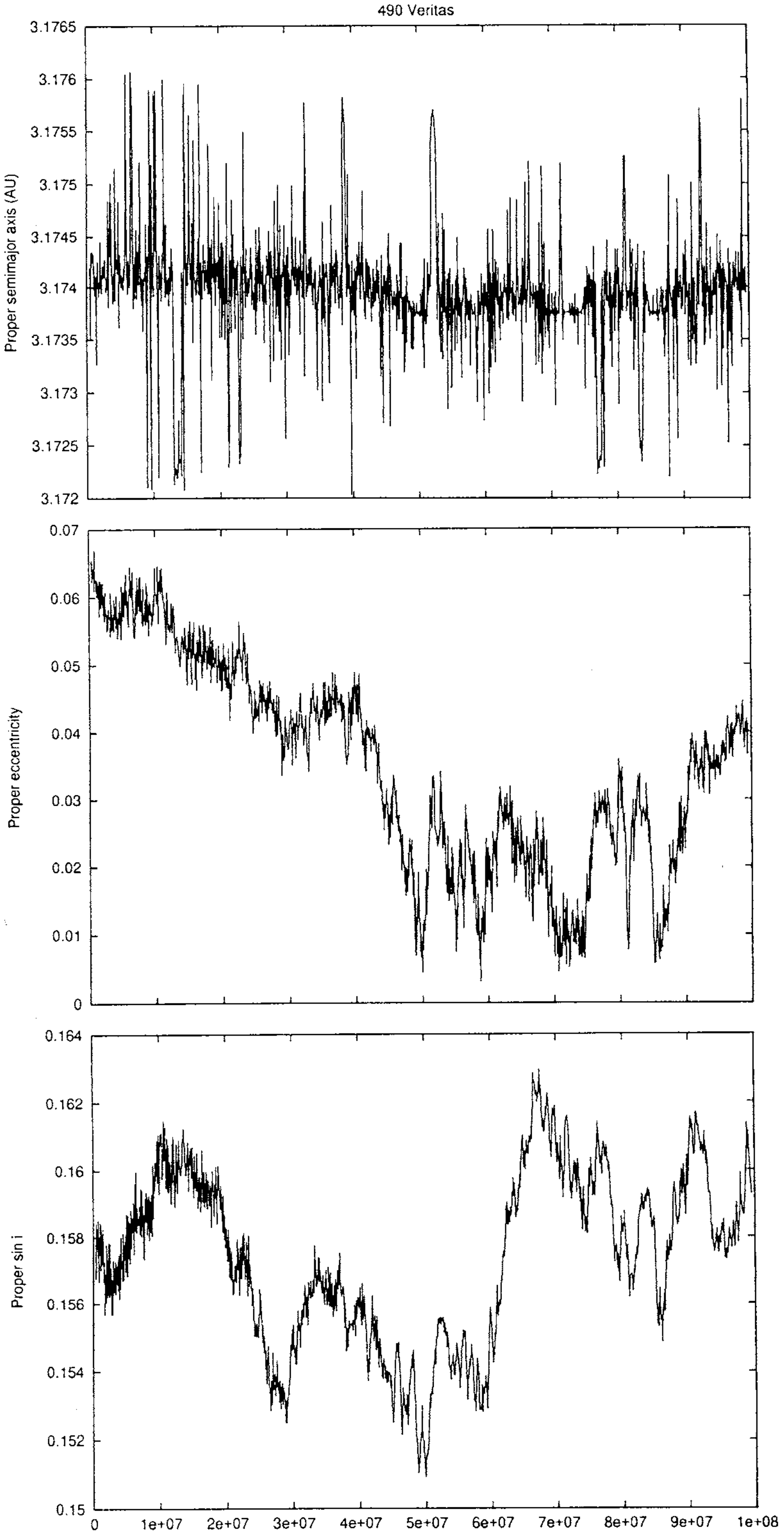

**Fig. 3b.** *Time evolution of the proper semimajor axis, proper eccentricity and proper sine of inclination for 490 Veritas (forward integration). Only very brief episodes of "capture" were observed in this case.*

Having thus identified the chaotic objects, we next performed for all seven of them numerical integrations covering 200 Myr (100 Myr in the past, and the same in the future), repeating essentially the same procedure we have already described for the previous step: on–line filtering of the short periodic perturbations, analytical computation of proper elements, and additional filtering to remove remaining perturbations of longer period. For this integration, however, the initial conditions were taken from the 1997 release of Bowell's orbital element data base.

In general, the results were as expected and confirmed the findings of the preliminary analysis, but they also revealed some new interesting features and phenomena, which shed a new light on the complex dynamics in this region. In the following we shall present our results on a body–by–body basis, trying at the same time to grasp some common behaviors and understand their causes.

Let us start from the major family member, that is 490 Veritas. In Fig. 3a we have plotted the time series of proper semimajor axis, proper eccentricity and proper sine of inclination over the 100 Myr backward integration of the asteroid's orbit, and in Fig. 3b the same has been done for the integration in the future. While the output of the forward integration exhibits only some well–known features of the motion of Veritas (chaotic wandering of the proper semimajor axis, combined with short episodes of temporary captures in high order mean motion resonances, and large variations of proper eccentricity and inclination; see Milani et al. 1996), the backward integration reveals a completely different behavior. After a brief period (a few million years) of instability, the body apparently gets trapped into the resonance (one of the harmonics involving mostly perihelia and few, if any, nodes), and stays there for the rest of the time. The semimajor axis variations become very small, and the proper $e$ and $\sin i$ remain quite stable for the entire integration time span (apart from a secular drift of small amplitude).

This latter result was certainly a kind of surprise, not so much because we found 490 Veritas trapped into the resonance, but because the trapping could last for so long. This is also an important result, as it might affect the estimate of the age of the family, based on the probabilistic arguments on a "fast" escape of the major body from the family region. In order to check the robustness of this result, we have performed a couple of integrations of clone orbits, obtained by changing the initial semimajor axis of the nominal orbit by plus or minus $10^{-7}$ AU. In both cases we found no trapping of the kind observed with the nominal orbit; as a matter of fact, we did observe temporary trappings, but lasting up to several millions of years, and not in the same resonance: one clone remained temporarily locked into another harmonic of the 21/10 resonance at a slightly larger semimajor axis, while the other stayed into the 44/21 resonance, at about 3.176 AU, for some 10 Myr (see Figs. 4a and 4b, where a similar behavior, but for another asteroid, is shown). It is therefore clear that the peculiar result we found with the nominal orbit is not necessarily common to a set of nearby orbits, but that similar, though not quite the same, phenomena do take place in its neighborhood; a tentative conclusion may be that the observed long–term resonant trapping might be just due to chance and to the essentially unpredictable nature of chaotic orbits in general. Later on, however, we found other examples of the same behavior, so we had to revise this conjecture.

The consequences for the problem estimating the family age are obvious: if family members which in some experiments drift away from the family zone after a given characteristic time, in other experiments stay inside the family due to a long term capture into the resonance, the age estimate obtained by M&F must be reconsidered. Any new estimate will be certainly valid only in probabilistic terms, and in any case more experiments are needed to derive such an estimate.

Figs. 4a and 4b are analogous to Figs. 3a and 3b, but show the time series of the proper elements for asteroid 3542 1964 $\mathrm{TN}_2$. In this case it is the orbit integrated into the past that exhibits large chaotic variations of all the three elements, with only a single very short capture into the resonance at about $-6.7 \times 10^7$ yr; the corresponding stabilizing effect against the chaotic changes can barely be seen in the plots of the proper eccentricity and proper sine of inclination. However, captures occurring in the forward integration lasted much longer, even up to 10 Myr, and the stabilization of the $e, \sin i$ elements in this case was much more effective. Also worth noting in this forward integration is the steady and fast increase of the proper eccentricity and proper sine of inclination in the entire integration time span, which brings this asteroid out of the family very quickly. This result might really serve as a paradigm for the well–known problems with orbital predictions based on the integrations of chaotic bodies. Even if qualitatively the integrations in the past and the future are supposed to give statistically similar results, in view of the time reversibility of the N–body problem, quantitatively these results can be quite different. Therefore, all conclusions based on these integrations are subject to a high degree of uncertainty, and only a large number of experiments with similar outcomes can be given enough credit.

The next couple of figures, 5a and 5b, shows the outcomes of our integrations for asteroid 5594 1991 $\mathrm{NK}_1$, which is processed in a slightly different manner with respect to the other ones. In this case, the plots for the eccentricity and inclination show their proper values as obtained by eliminating the perturbations up to 100,000 yr of period; this because in this case perturbations with shorter periods completely masked the underlying chaotic behavior, which could be revealed only by applying another, more appropriate filter.

The results confirmed what was expected on the basis of the previous results, as discussed above. Again captures of different durations and into different resonant harmonics were observed in the proper semimajor axis, associated with episodes of quasi–stable behavior in proper $e, \sin i$. In agreement with the prediction of Milani et al. (1996), this asteroid, which is located in the zone of the 21/10 mean motion resonance populated with harmonics involving both perihelia and nodes, exhibits a typical stable

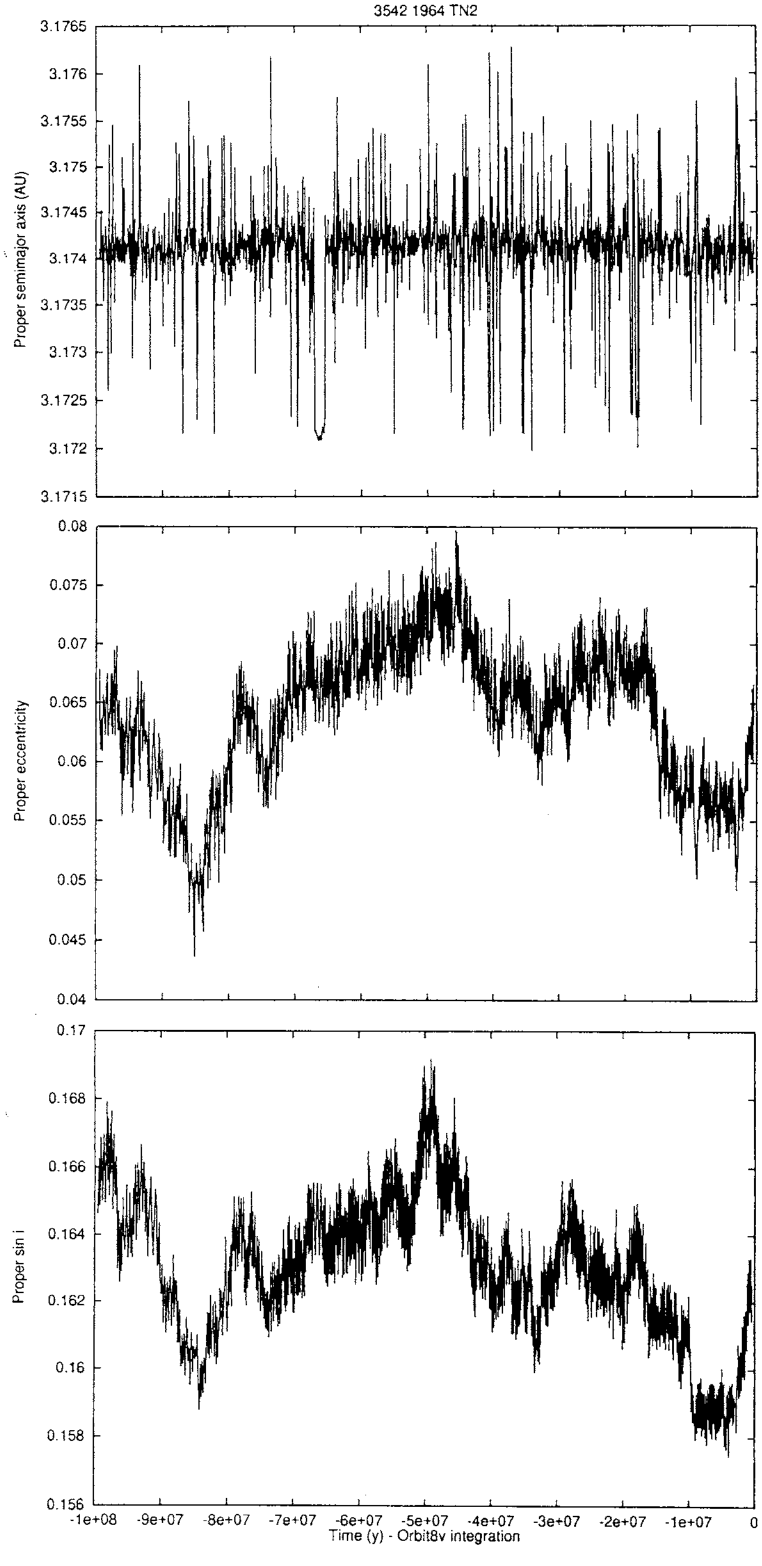

**Fig. 4a.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 3542 1964* $TN_2$ *(past). The large variations of the proper elements are a primary feature of this experiment.*

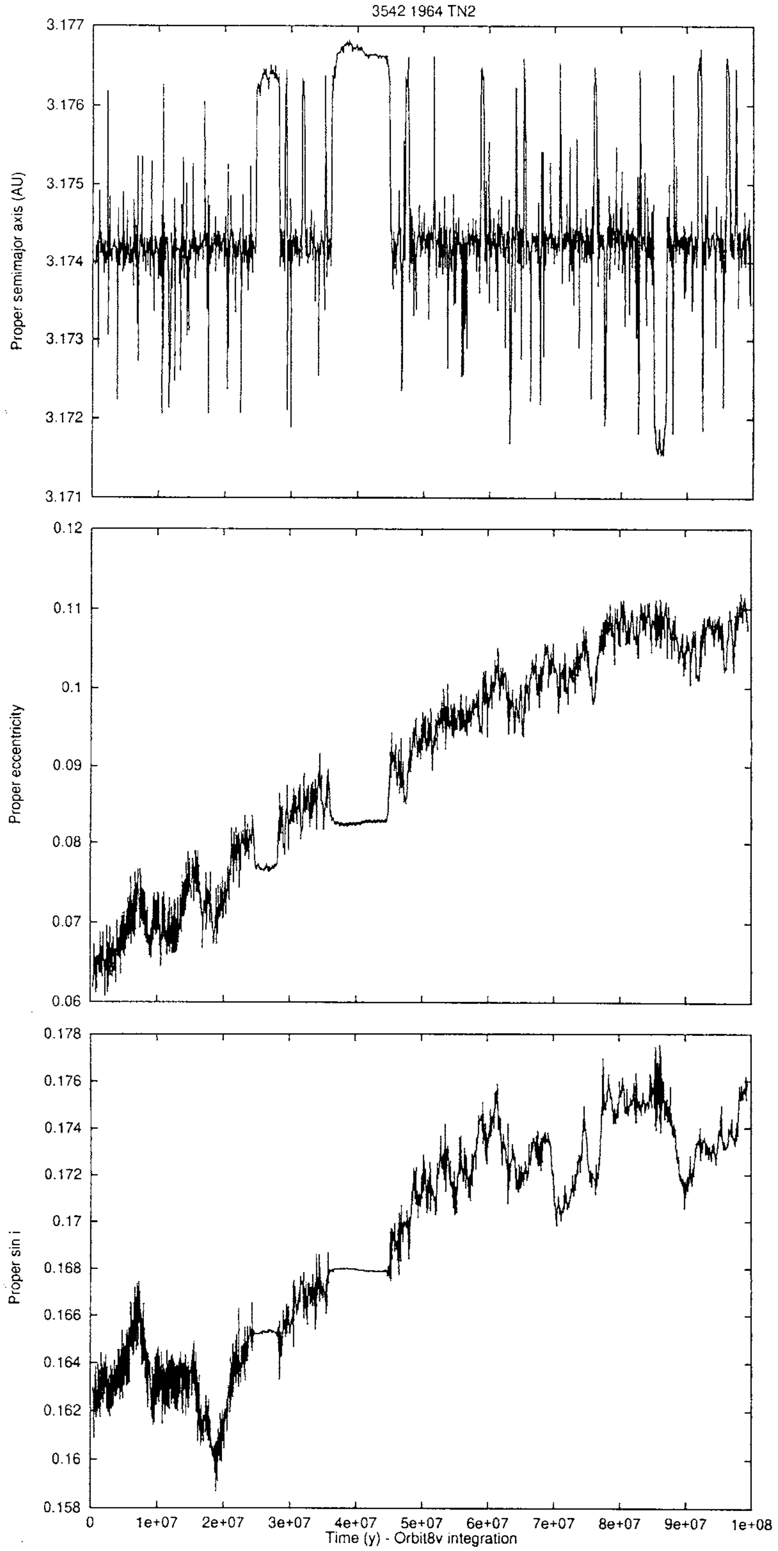

**Fig. 4b.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 3542 1964* $TN_2$ *(future). Capture into different resonances occurs, lasting up to 10 Myr.*

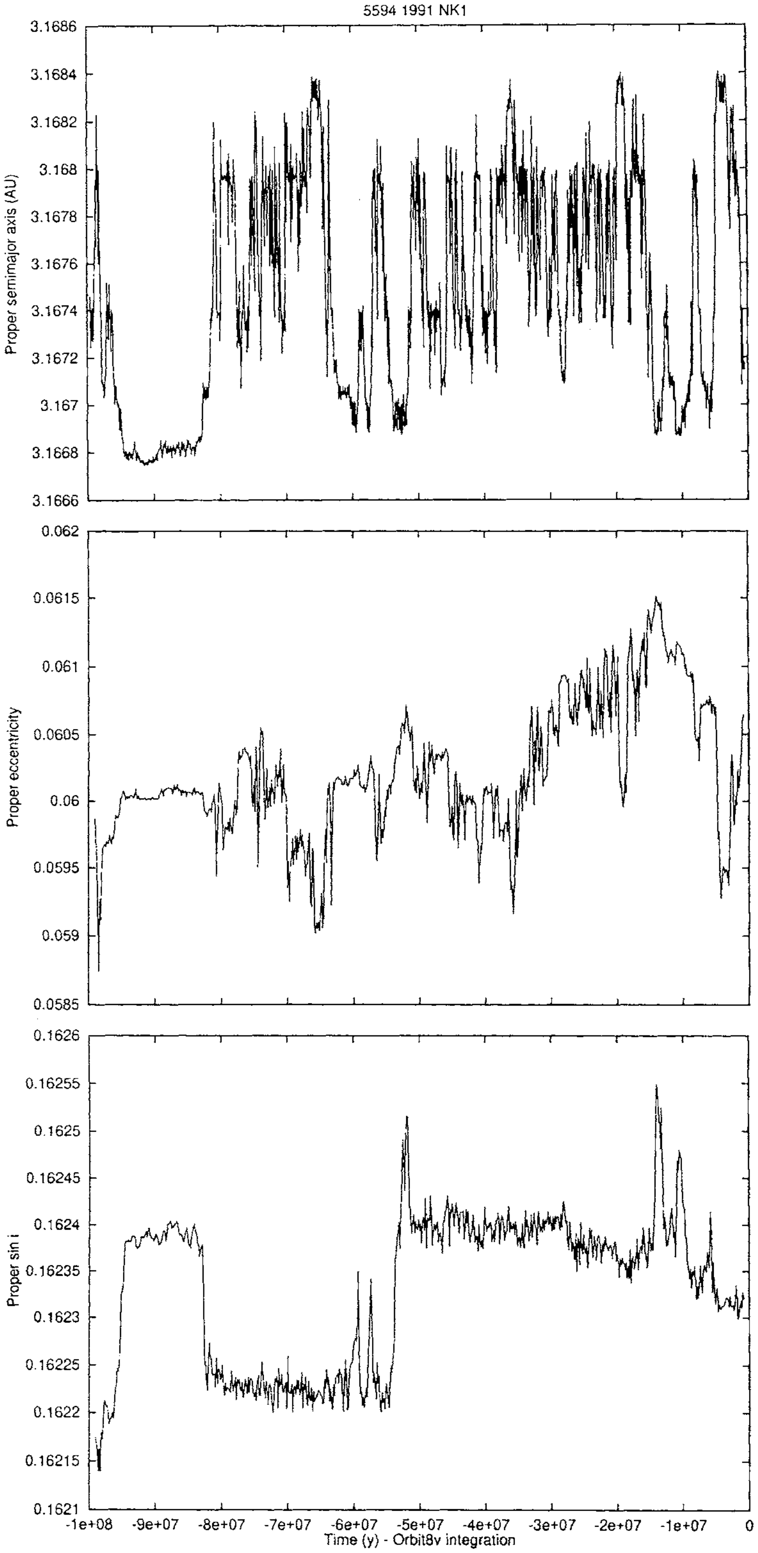

**Fig. 5a.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 5594 1991* $NK_1$ *(past). The jumps in inclination are associated with captures into a resonance located at 3.167 AU*

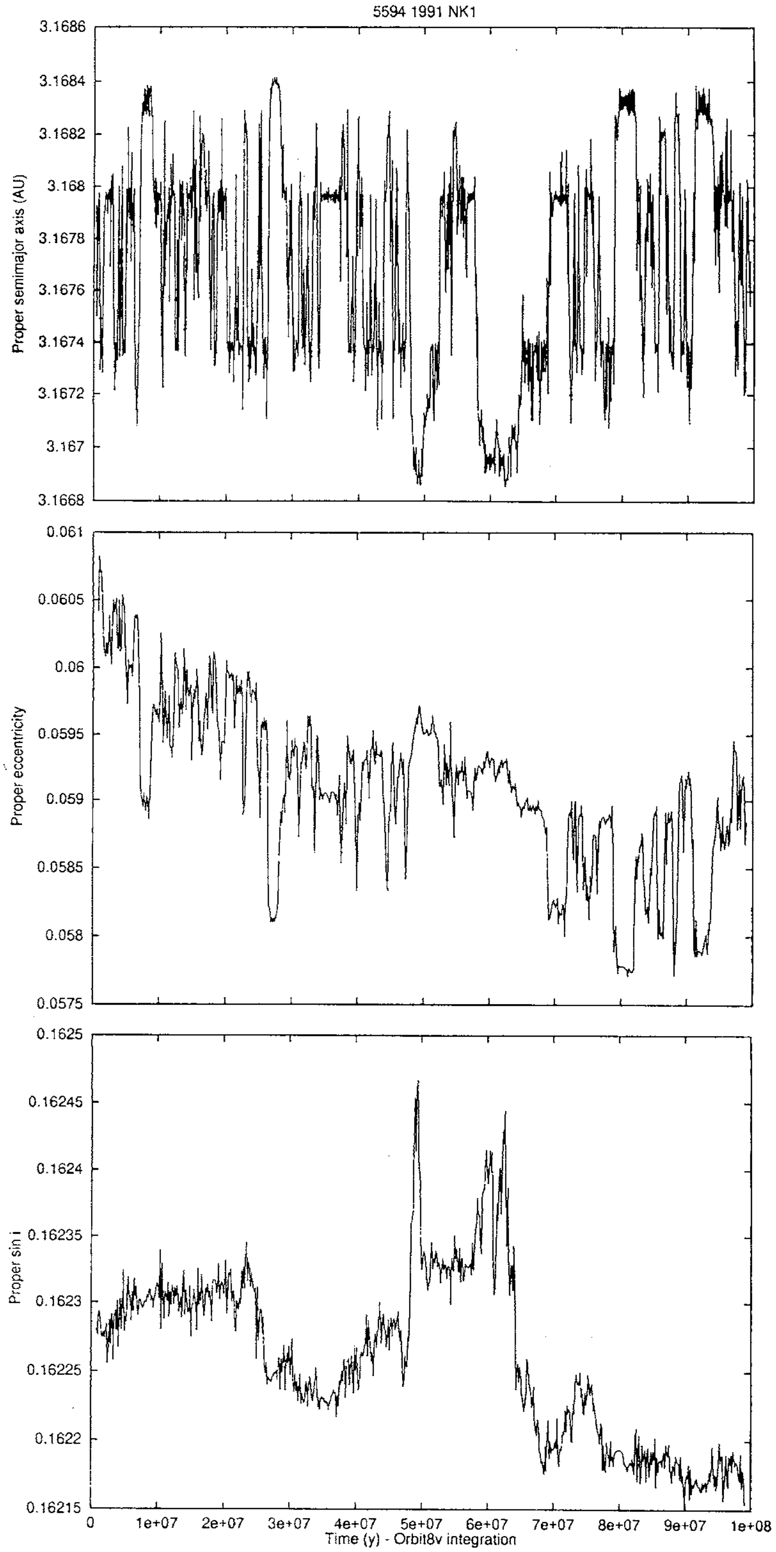

**Fig. 5b.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 5594 1991* $NK_1$ *(future).*

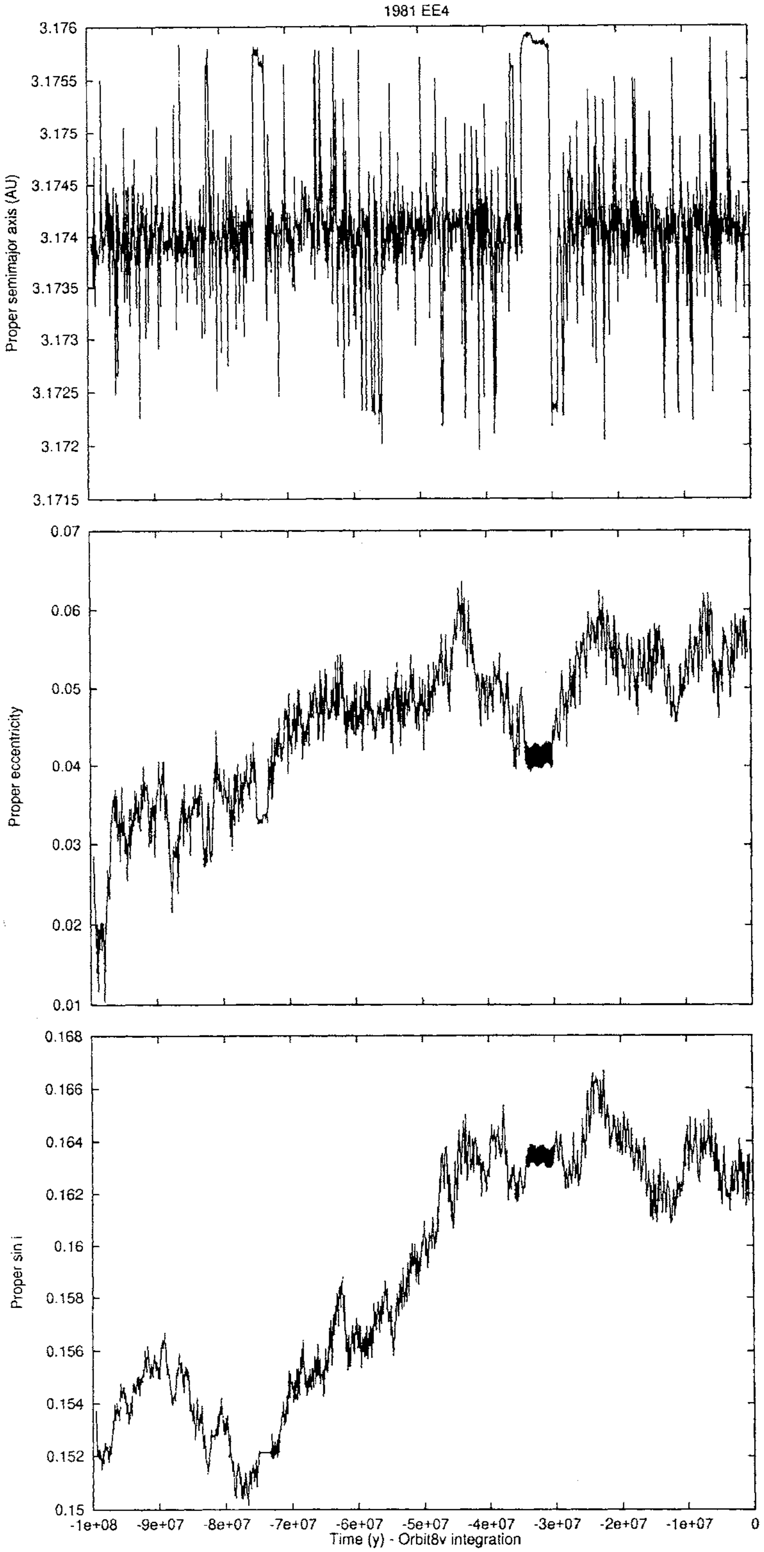

**Fig. 6a.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for the single–opposition body 1981 $EE_4$ (past). The proper e and proper* $\sin i$ *exhibit a long periodic trend.*

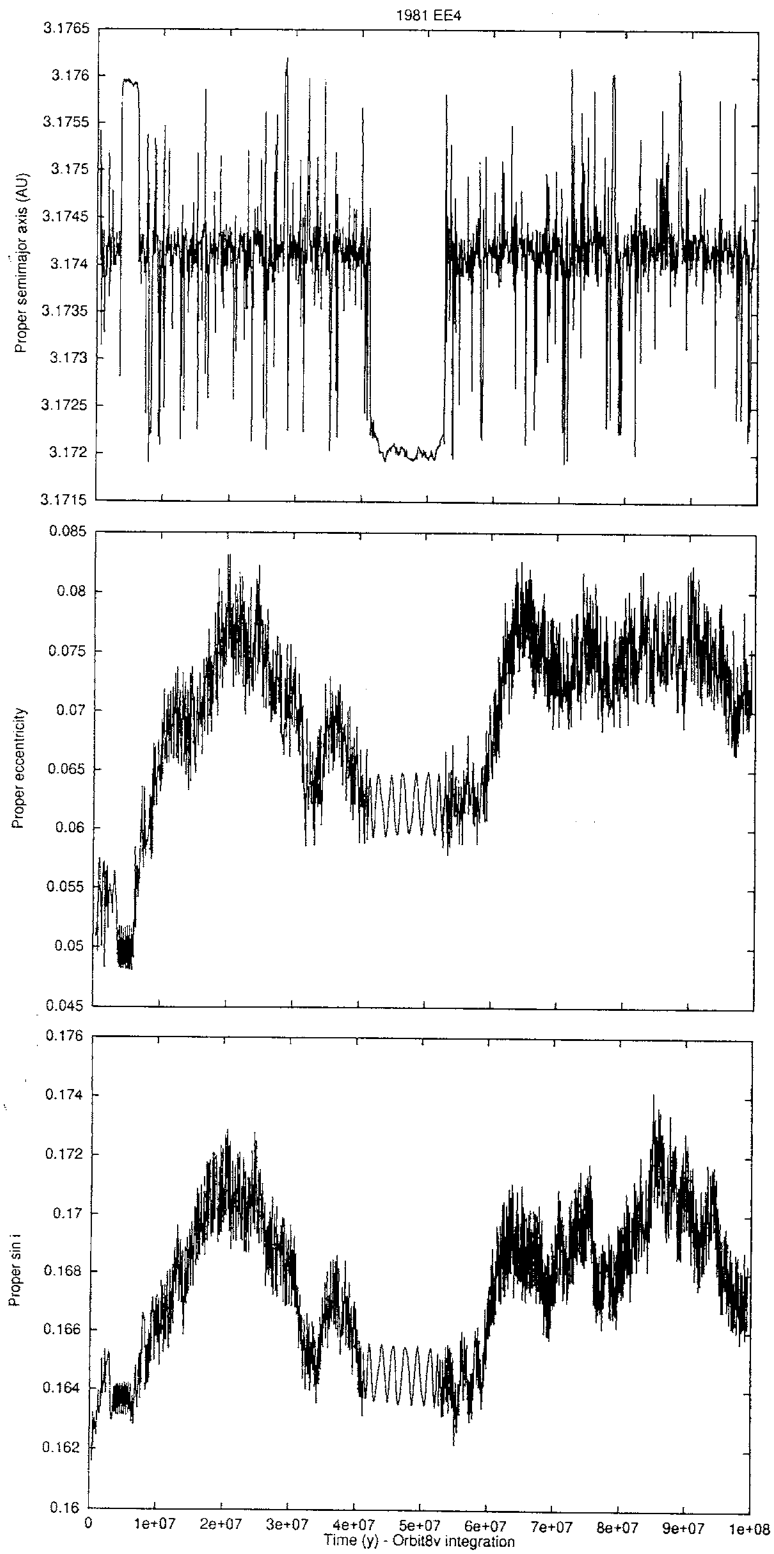

**Fig. 6b.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 1981 $EE_4$ (future).*

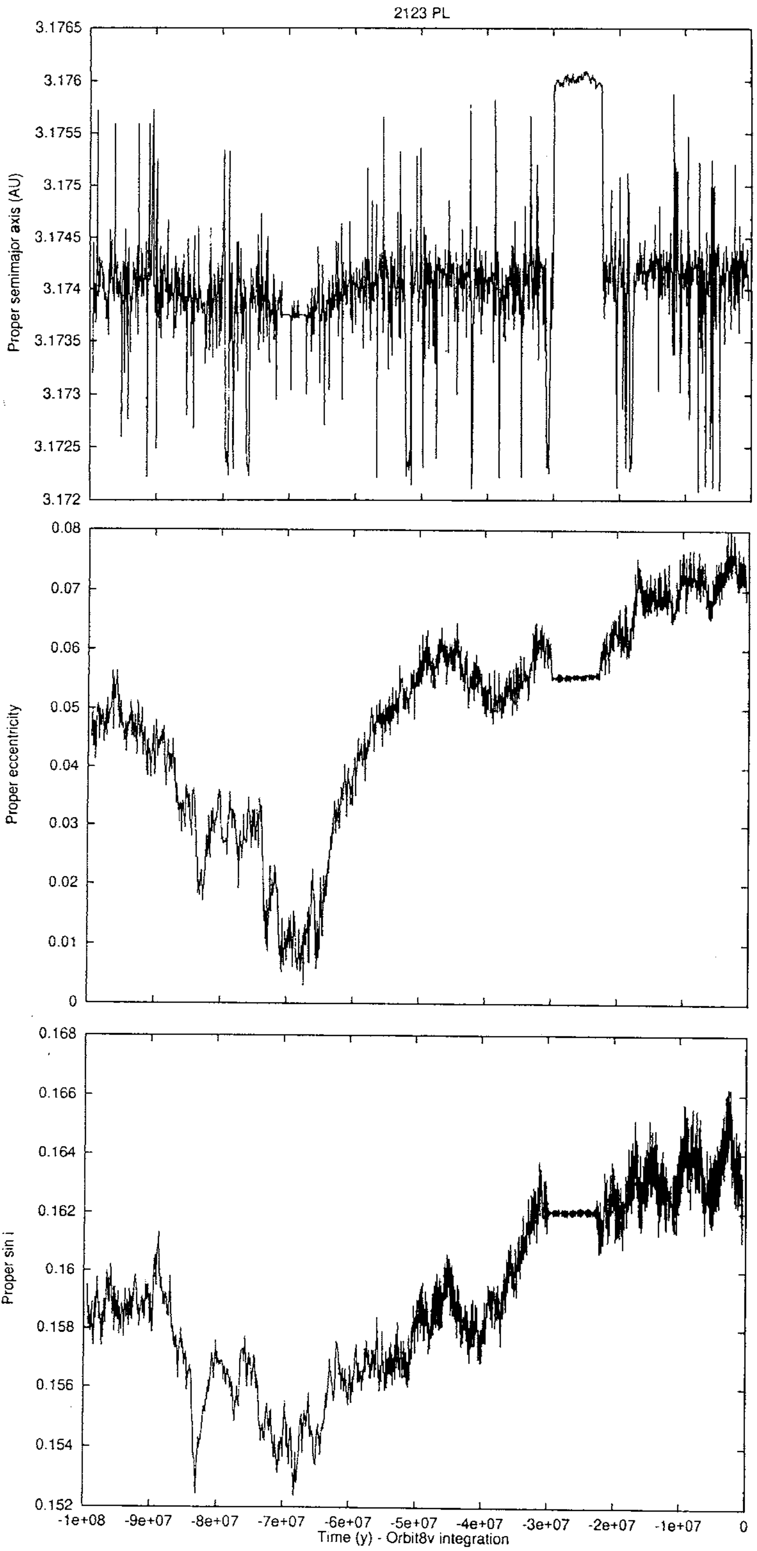

**Fig. 7a.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for the single–opposition body 2123 PL (past).*

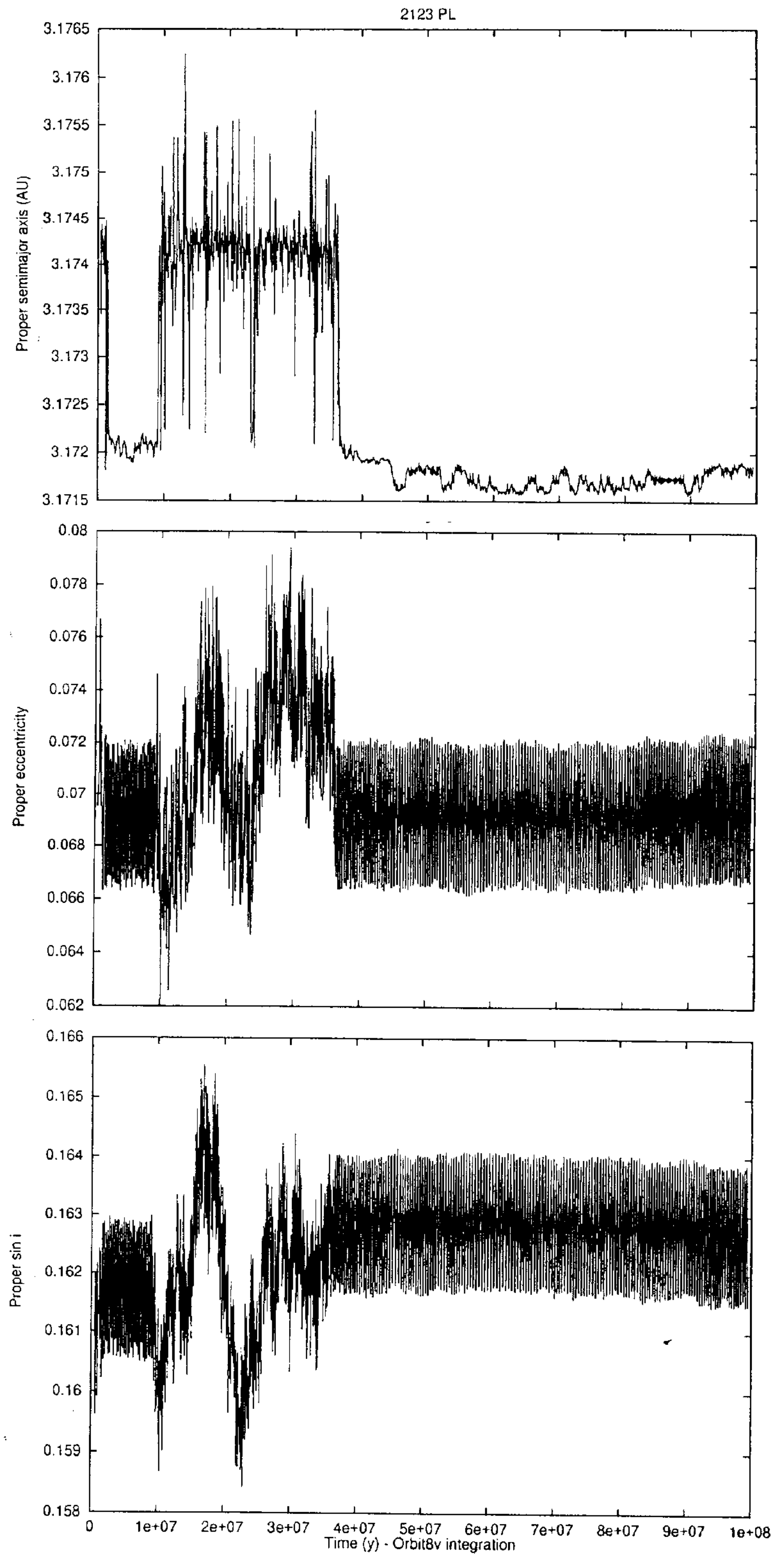

**Fig. 7b.** *Proper semimajor axis, proper eccentricity and proper sine of inclination for 2123 PL (future). Note the capture in the same resonance harmonic in which 490 Veritas remained also locked for almost 100 Myr.*

chaos behavior, but with variations of smaller amplitude and with longer Lyapunov time. However, it is puzzling that this body appears to be, in a way, "too chaotic", (more chaotic than 2147, for example, while the opposite should be expected according to Milani et al. 1996), again pointing out that the dynamics in the region of Veritas must be more complex than it might have been expected. It can be easily verified that the jumps in the proper sine of inclination visible in Figs. 5a and 5b are associated with captures into a resonance located at 3.167 AU, which is exactly where the 40/19 mean motion resonance occurs, so that the amplified chaos might be due to an overlapping of the harmonics of the two resonances.

There is not much to say about 1981 $EE_4$. The results are shown in Figs. 6a and 6b, and they are qualitatively the same as those for 3542, for example. One notices a kind of decreasing trend of the proper eccentricity and proper sine of inclination in the backward integration, large enough to bring this asteroid out of the family. Again there are temporary captures into resonances, and so on.

The results for 2123 PL are more interesting, in particular those for the forward integration. While the backward integration (Fig. 7a) reveals again the typical behavior observed in other cases, the forward one (Fig. 7b) shows another example of a long–term capture, lasting more than 60 Myr, into the same resonance in which 490 Veritas has been found to stay trapped. Thus, these long trappings do not seem to be so exceptional, and the above cautionary conclusions on the estimate of the family age are confirmed (see also later).

Another interesting point about these results pertains to a shorter ($\simeq$ 8 Myr) locking, into a different resonant harmonic at a slightly higher semimajor

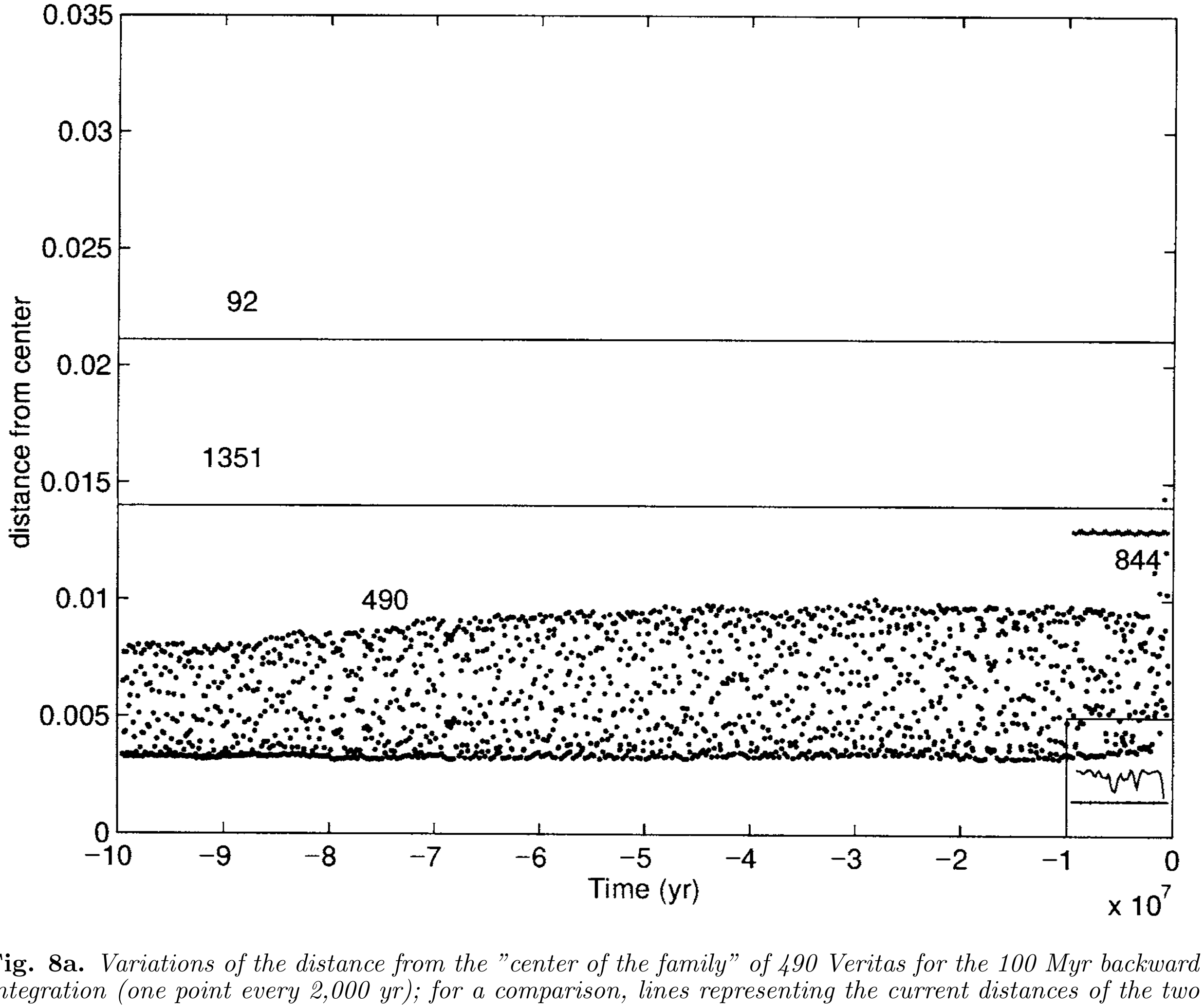

**Fig. 8a.** *Variations of the distance from the "center of the family" of 490 Veritas for the 100 Myr backward integration (one point every 2,000 yr); for a comparison, lines representing the current distances of the two background objects 92 and 1351, as well as the 10 Myr variations of the distance of 844 Leontina, are also given. The small square in the lower right corner of the figure shows the 10 Myr variations of the distance for the two stable family members (2147, upper curve, and 1086, lower one) located near the family center. Apart from the very brief excursion at the beginning of the integration, Veritas remains within the family boundaries for all the time.*

axis (see Fig. 7b); this locking is also associated with a stabilization of the chaotic oscillations of the proper inclination, but at a somewhat lower value than for the above mentioned long–term locking (proper eccentricities, on the contrary, in both cases remained about the same on the average). Note that the harmonic responsible for the long–term capture, being located at a lower semimajor axis, involves more nodes than the one responsible for the shorter capture (see Milani et al. 1996), and that this might be the reason for the observed difference.

The results for 4573 PL and its clones did not reveal anything new, while 1118 T3 proved to be another case of long–term capture in the resonance, lasting this time for some 90 Myr, while its clones behaved more regularly.

Note that for the majority of the considered cases, the evolution of proper elements in the $(e, \sin i)$ plane (see Fig. 2) sooner or later brings a given asteroid out of the family boundaries. Exceptions to this are cases in which long–term captures occurred, and the single less chaotic body (5594); even these orbits, on the other hand, wander around creating characteristic "clouds" in the phase space, but they did not exit from the family zone in the covered time span.

The final result to be considered here pertains to the variation in the proper element space of the distances between chaotic family members in the phase space of proper elements, or between a given member and the "center of the family". In deriving these distances in terms of the $d_1$ metrics of Zappalà et al. (1995), we used as origin an arithmetical mean of the 10 Myr averages of proper elements of the five most stable members of the family (1086, 2147, 2428, 2934, 3090). Note that this is a slightly different origin with respect to that used by M&F (the position of 1086 itself), but this discrepancy has no practical importance. In Fig. 8a we show the variations of the distance of 490 Veritas for the 100 Myr backward integration; for a comparison, the lines representing the current distances of two background objects (asteroids 92 and 1351), as well as the 10 Myr variations of the distances of the newly included member of the family 844 Leontina, are plotted too. These three lines are there to provide an idea of the limiting dis-

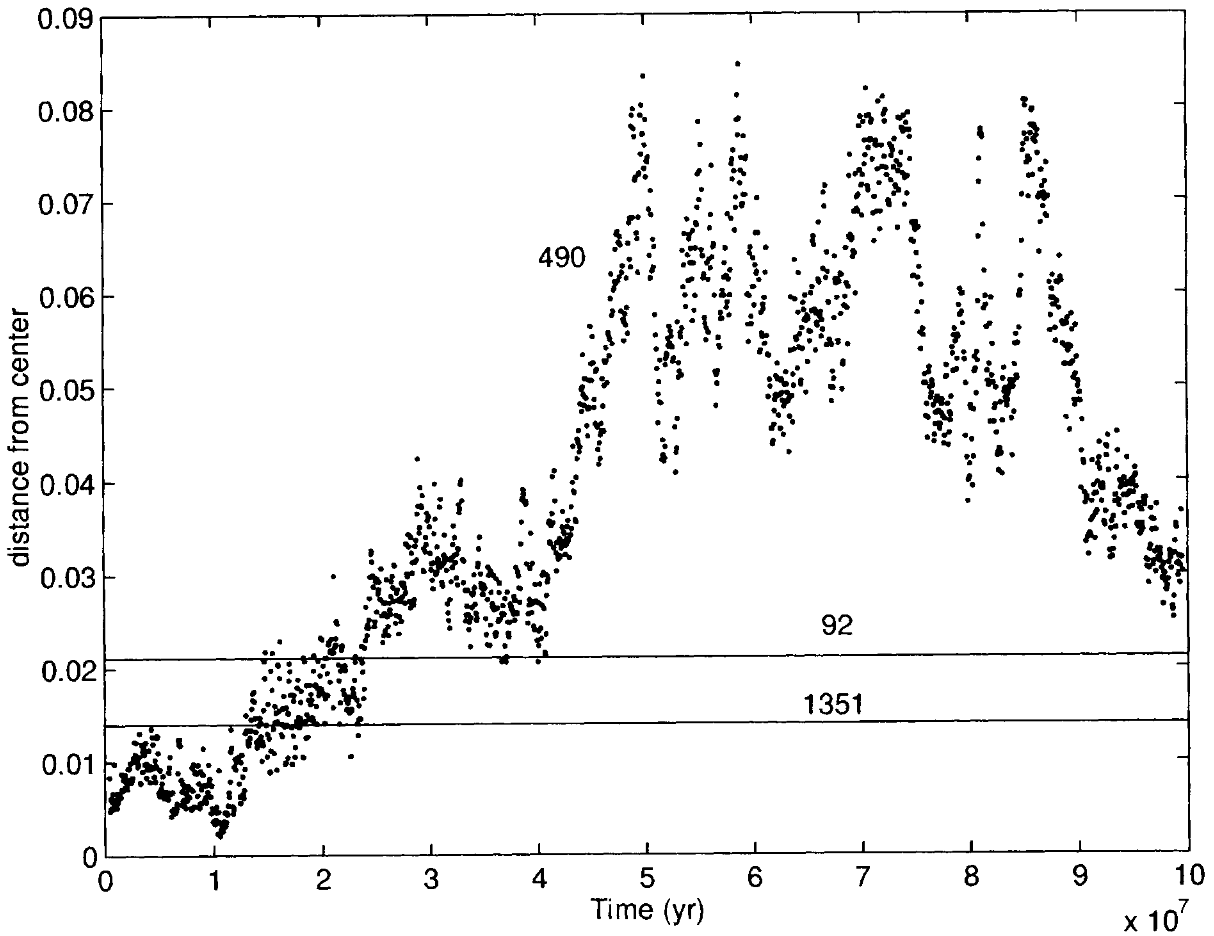

**Fig. 8b.** *Variations of the central distance of 490 Veritas for the 100 Myr integration in the future. The body escapes from the family zone already after some 10 Myr.*

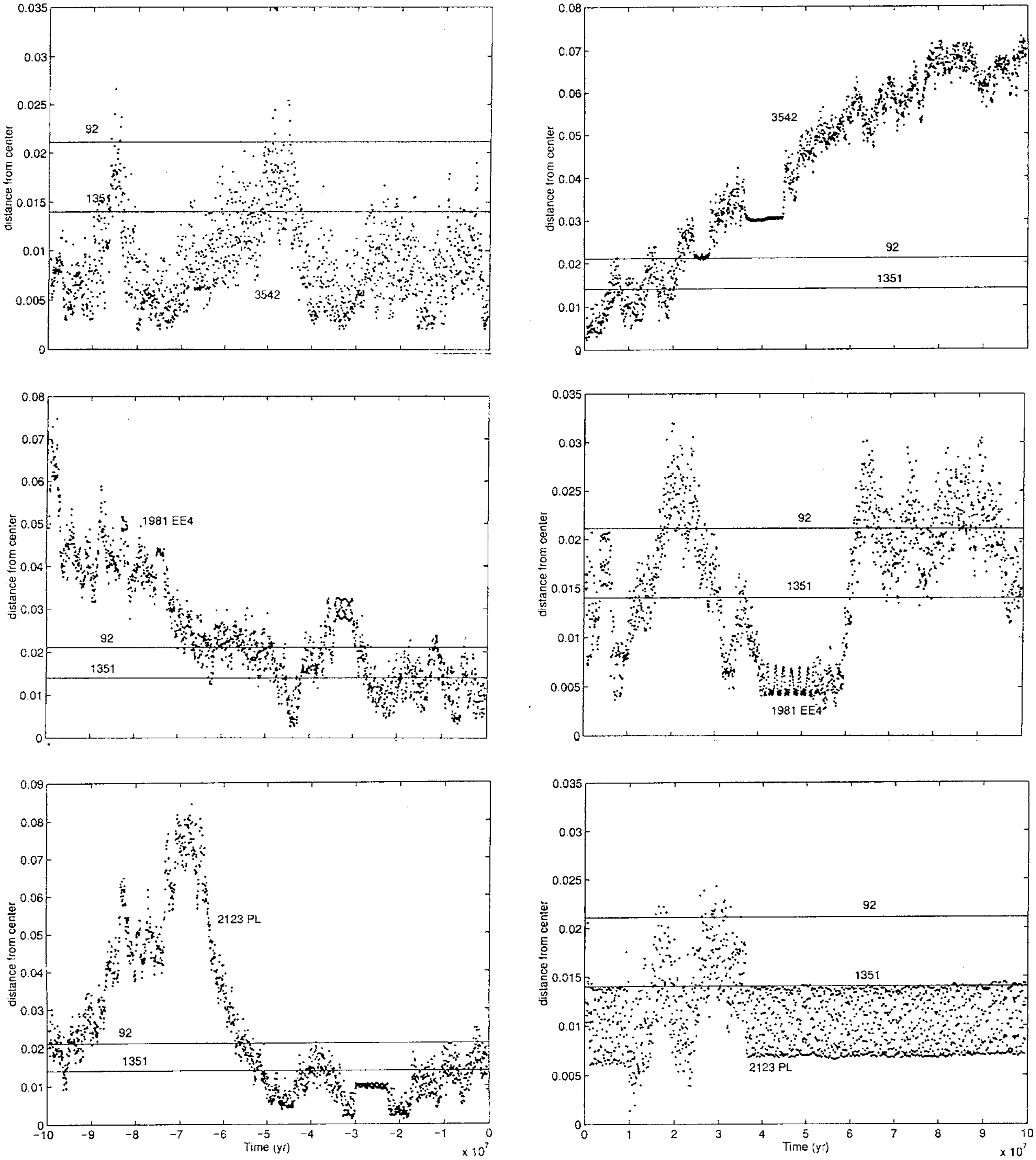

**Fig. 9.** *A composite figure showing the variations of the distance from the "center of the family" for three chaotic members of the Veritas family (*3542 1964 $TN_2$*, 1984 $EE_4$ and 2123 PL). On the left–hand side, the results coming from the 100 Myr backward integration are shown, and on the right–hand side the results from the forward integrations (note the different vertical scales in the plots).*

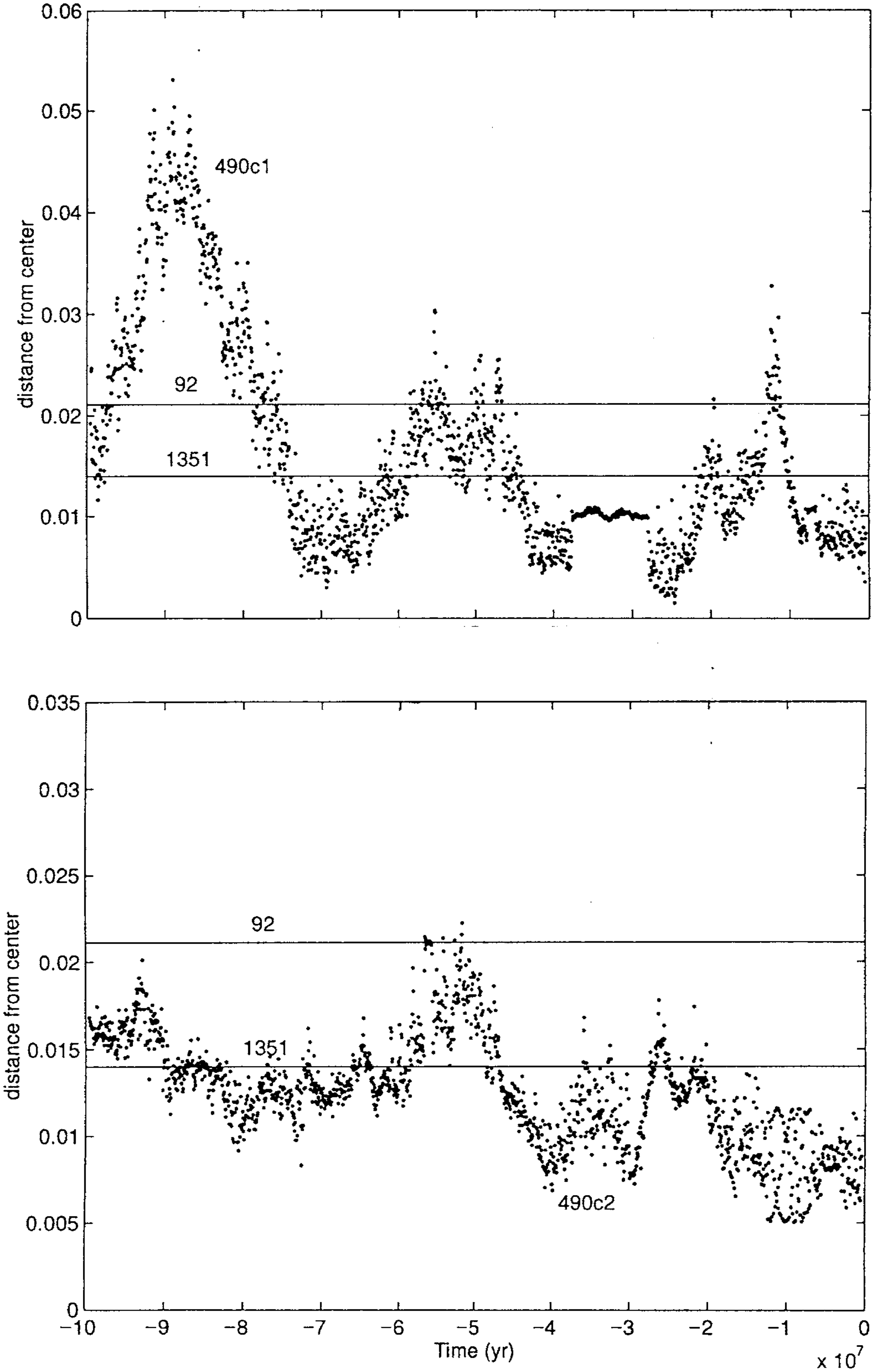

**Fig. 10.** *Variations of the central distance of the two clones of 490 Veritas, for the 100 Myr integration in the past.*

tance for the family members: obviously the family border should be somewhere between 0.014 and 0.015 in the arbitrary units of the metrics used in defining distances. In addition, a small square in the lower right corner of the figure shows again the 10 Myr variations of the same distance, but for the two stable family members (2147, upper curve, and 1086, lower one) located near the family center. One can readily conclude from the plot that, apart from the very brief excursion at the beginning of the integration, Veritas (due, of course, to the capture into the resonance) remains within the family boundaries for all the time span. On the contrary, Fig. 8b, in which the future variations of Veritas' distance are presented, shows a completely different behavior, that is, a fast increase of the distance and the escape from the family zone in as little as $\simeq 10^7$ yr. In both cases, these results are very different from those obtained by M&F, and again point out that the previous results should be considered as preliminary and the conclusions as uncertain, at least until more data are produced and analyzed.

The results for three other chaotic bodies with reliable orbits are assembled in Fig. 9, where for each of them the variation of the distance is shown for the forward and the backward integrations. A remarkable feature is the significant past vs. future asymmetry of the results. Moreover, the three future integrations produced quite different outcomes: a fast increase of the distance and the escape from the family for asteroid 3542, a wandering across the family boundary for 1981 EE4, and a similar behavior, but affected by the resonant capture, for 2123 PL; this latter body, due to the capture effect, spends most of the time inside the family borders.

On the contrary, the integrations in the past show for 3542 a typical stable–chaos random wandering of the distance across the family border and episodes of temporary escapes and returns, while for the other two asteroids the behavior is very much the same as that found by M&F in their study: a chaotic wandering similar to that just described for 3542, followed by a sudden and fast increase of the distance, occurring between 50 and 60 Myr from the present, and eventually the escape from the family (but see an example of a return after a long "absence" in the plot for 2123 PL at about $-9.7 \times 10^7$ yr). This latter result is confirmed by the results for the two clones of Veritas. As seen from Fig. 10, the variations of the distances for these two bodies behave in a very similar way.

## 3. DISCUSSION AND CONCLUSIONS

The results presented in this paper are interesting from several points of view and represent an important extension of the results of M&F. First of all, there are many more data for a detailed analysis of the dynamics in the chaotic region, since these data are produced by integrations covering a much longer total time span and a larger number of chaotic orbits have been integrated. All this provides a new insight into the complex phenomena taking place in the Veritas family and reveals some new features of the family itself, as well as of a number of its members.

One of the novel findings of the present study is the discovery of a number of previously unknown chaotic members of the family. As we pointed out earlier, these bodies were recognized as members of the family only recently, together with some other newly added, but more stable, objects. Their orbits are not always accurate, but in most cases they are accurate enough for the purpose of a qualitative study of their dynamical evolution. All the strongly chaotic asteroids are located close to each other in semimajor axis, on the right–hand side of the 21/10 mean motion Jovian resonance, and all of them have Lyapunov times on the order of $10^4$ yr, in agreement with predictions of Milani et al. (1996).

The most important result of this analysis pertains to the estimate of an upper limit for the age of the Veritas family. On one hand, we have found that for the majority of the chaotic bodies the integrations in the past reveal a behavior similar to that found and used by M&F to establish the $\approx 50$ Myr upper bound for the family age; all the bodies exhibit a characteristic stable chaos behavior for several tens of Myr, then suddenly begin to put distance between themselves and the rest of the family, and eventually leave the region in the phase space occupied by the family. This behavior strengthens the conclusion of M&F on the relatively young age of the family, and in particular their probabilistic arguments based on the ergodic principle seem to be fully applicable in this situation. Moreover, the probability that all the escaping members could have found themselves together and near the other members at some remote epoch(s) in the past (and that the family was formed during one of these collective chance returns) appears to be negligible.

On the other hand, the integrations in the future revealed a somewhat different picture. More specifically, one can say that the qualitative picture is roughly the same as that for the backward integrations, but that in the forward integrations the distances with respect to the family core increased in some cases much faster than for the experiments in the past, bringing these bodies out of the family in less than few tens of Myr. This is in agreement with what we already stated above on the time reversibility of the N–body problem, and the essentially unpredictable nature of chaotic motions, and obviously brings in a bit of uncertainty when the age of the family is in question. This uncertainty is enhanced by the long term resonant captures which we observed in a number of experiments, including those for the largest family member 490 Veritas: were this phenomenon very common, it would be conceivable the family to be much older than inferrred from the probabilistic argument of M&F. However, this is probably not the case: considering only the four bodies which have accurate osculating orbits and might exit the region occupied by the family, our integrations covered a total of some 1.9 Gyr of chaotic orbital evolution and we found long term captures for three bodies, lasting some 250 Myr overall; adding also the short term captures in different resonances, we estimate that there is a probability of some 15 – 20 % for any body in this chaotic region to be at any time locked in a resonance, and stay temporarily in a quasi–stable state within the family borders. If this estimate is correct, it would be rather unlikely (say, a $(0.2)^4 \approx 1.6 \times 10^{-3}$ probability) that all the chaotic family members at any given time (e.g., now) were staying in one of these quasi–stable states. Therefore we conclude that it is also unlikely (though not impossible) that the family age is longer than 50–100 Myr.

Various types of dynamics are found to coexist in the Veritas family region. These diverse behaviors range from a remarkable stability to a strong chaos. Typically, chaotic bodies jump between resonant harmonics and the resulting variations of the (proper) orbital elements are large and entirely unpredictable; there are also regions in the phase space where the dynamical evolution has all the characteristics of chaotic motion, but the elements are nevertheless stable for very long times. This just highlights the extraordinary complexity of this region from the dynamical point of view, once again stresses the importance of the high order mean motion resonances and their effects, and emphasizes the need to investigate these interesting phenomena in more detail. We plan to work in this direction in the near future.

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# ДА ЛИ ХАОТИЧНИ ЧАСОВНИК ТАЧНО ОТКУЦАВА?

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У раду је приказана обимна анализа динамике свих познатих чланова астероидне фамилије Веритас. Анализа је урађена у циљу провере процене старости ове фамилије, коју су дали Milani и Farinella (1994) користећи методу тзв. "хаотичне хронологије". Ми смо кренули од знатно већег узорка чланова фамилије и извршили далеко већи број интеграција путања свих њених чланова, а нарочито тела са хаотичним кретањем; интеграције за ове последње су обухватиле по 200 милиона година и покриле укупно око 2.1 милијарду година. Показало се да је динамика у области фазног простора елемената кретања коју заузима фамилија Веритас комплекснија него што се то раније веровало и да ту постоје тела са кретањем у распону од изразите стабилности до потпуног хаоса; хаос је узрокован утицајем резонанце у средњем кретању 21/10 са Јупитером и њеним хармоницима који укључују споре угловне променљиве. Нађено је укупно седам изразито хаотичних тела, која показују слично понашање у квалитативном смислу, а у складу са предвиђањима Milani-ја и др. (1996), али такође и извесне до сада непознате особине. Од новооткривених особина најважнија се односи на чињеницу да неки од хаотичних астероида, укључујући и највеће тело у фамилији, сам астероид 490 Веритас, могу бити ухваћени у резонанцу и остати ту врло дуго у квази-стабилном стању. Иако наши резултати углавном нису у супротности са закључцима Milani-ја и Farinella-е и добро се слажу са њиховом проценом старости фамилије, постоје и неки интересантни резултати ове анализе који отварају нова питања и захтевају свеобухватнију и детаљнију анализу.