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SOME CORRELATIONS FOR MASSIVE MS STARS

T. Angelov

Astronomical Institute, Faculty of Mathematics, Studentski trg 16, 11000 Belgrade, Yugoslavia

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SUMMARY: Criteria are derived for estimating the values of photospheric density, of the core mass and of the energy-generation rate for massive main-sequence stars. Based on the observational material concerning $\mathcal{M}$, L and T_e the demarcation lines are determined for the domain of values expected for these quantities in correlation with measured M_b.

1. INTRODUCTION

The empirical HR diagram enables testing to be made of the theory of stellar structure and evolution. For this purpose various, mostly two–parameter, correlations are derived between observed stellar parameters. The most important feature of the diagram is its main sequence (MS). It is interpreted as a region of stars (various masses, chemical composition and age) with hydrogen reactions in the core. The empirical mass–luminosity relation for different parts of the main sequence is most widely used. Therefore, a high accuracy in the measured spectral characteristics and masses for MS components of various double-star types, especially with $\mathcal{M} >> \mathcal{M}_\odot$ is of particular interest.

In this paper is presented a procedure for the purpose of evaluating the photospheric density, the core mass and the nuclear-energy-generation rate for the stars from the upper branch of the main sequence ($\mathcal{M} > 2\,\mathcal{M}_\odot$). The criteria for the limiting values of these quantities are derived in Section 2. The next step (Sect. 3) is to determine the demarcation lines for the domains of possible values for $\rho_{\rm ph}$, $\bar{\varepsilon}_{\rm core}$ and $\mathcal{M}_{\rm core}$ in correlation with M_b. For this purpose is utilised Popper's review (1980) of $\mathcal{M}$, L and T_e measured for MS stars.

2. BASIC RELATIONS

Main-sequence stars with $\mathcal{M} > 2\,\mathcal{M}_\odot$ without rotation and magnetic field will be considered. Their interior will be approximatively represented by a convective core and an envelope in radiative equilibrium. For the ideal gas with black-body radiation the following variable will be introduced

$$x(r) = \frac{P_{\rm rad}}{P_{\rm gas}} \ ; \quad P_{\rm rad} = \frac{a}{3}T^4 \ , \quad P_{\rm gas} = \frac{\mathcal{R}_g}{\mu}\rho T \ , \tag{1}$$

where the standard designations for the universal constants, density, temperature, pressure and the mean molecular weight are used. Since $P = P_{\rm gas} + P_{\rm rad}$, in view of (1) it will be

$$T(x,\rho) = K_1 x^{1/3}\rho^{1/3} \,,$$
$$P(x,\rho) = K_2(1+x)x^{1/3}\rho^{4/3} \,,$$
$$P(x,T) = (a/3)\frac{1+x}{x}T^4 \,, \quad (2)$$

where

$$K_1 = (3\mathcal{R}_g/a)^{1/3}\mu^{-1/3}, \quad K_2 = (a/3){K_1}^4 \,. \quad (3)$$

Further, a homogeneous chemical composition throughout the stellar interior will be considered.

In a special case, for adiabatic variations, from the first law of thermodynamics

$$[1+12(\gamma-1)x]\frac{\mathrm{d}T}{T} - (\gamma-1)(1+4x)\frac{\mathrm{d}\rho}{\rho} = 0 \,,$$

by using (2) one derives the integrated adiabates

$$\rho(x) = C_1 x^{\lambda/(\gamma-1)} e^{12\lambda x},$$
$$T(x) = C_2 x^{\lambda} e^{4\lambda x},$$
$$P(x) = C_3(1+x)x^{\lambda\gamma/(\gamma-1)} e^{16\lambda x}. \quad (4)$$

Here $\lambda = (\gamma-1)/(3\gamma-4)$, $\gamma = c_P/c_V$ for the ideal gas, whereas C_1, C_2, C_3 are the integration constants (see also Chandrasekhar, 1951; Menzel *et al.*, 1963).

For the convective core the adiabatic approximation will be used. Therefore, in view of the criterion for the choice of the energy-transfer mechanism, we shall use the quantities $\nabla_{\rm rad}$ and $\nabla_{\rm ad}$, where $\nabla \equiv \mathrm{d}\ln T/\mathrm{d}\ln P$. For the stellar interior in hydrostatic (and thermal) equilibrium, from

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -\frac{G\mathcal{M}(r)}{r^2}\rho \quad (5)$$

and

$$\left(\frac{\mathrm{d}T}{\mathrm{d}r}\right)_{\rm rad} = -\frac{3\kappa\rho}{4acT^3}\frac{L(r)}{4\pi r^2} \,,$$

by using (1) one obtains

$$\nabla_{\rm rad} = \frac{\kappa\eta}{C_*}\frac{1+x}{x} \,. \quad (6)$$

Here

$$C_* = \frac{16\pi cG\mathcal{M}}{L} \,, \quad \eta = \frac{\bar{\varepsilon}(r)}{\bar{\varepsilon}(R)} = \frac{l}{q} \,,$$
$$l = \frac{L(r)}{L} \,, \quad q = \frac{\mathcal{M}(r)}{\mathcal{M}} \,, \quad \bar{\varepsilon}(r) = \frac{L(r)}{\mathcal{M}(r)} \,, \quad (7)$$

where $\mathcal{M}$, R and L are the mass, the radius and luminosity of the star, $\kappa(r)$ is the opacity, $\varepsilon(r) \equiv \varepsilon_{\rm nuc}(r) = \mathrm{d}L(r)/\mathrm{d}\mathcal{M}(r)$. On the other hand, by using the adiabates $T(x)$ and $P(x)$ from (4) one obtains

$$\nabla_{\rm ad} = \frac{(1+x)(1+4x)}{16x^2+20x+\gamma/(\gamma-1)} \,. \quad (8)$$

Everywhere within the radiative envelope, including also the surface layer (but not within the core), it $\nabla_{\rm rad} < \nabla_{\rm ad}$ is valid which yields according to (6) and (8)

$$\kappa < C_*\frac{f(x)}{\eta} \quad (9)$$

with

$$f(x) = \frac{x(1+4x)}{16x^2+20x+\gamma/(\gamma-1)} \,. \quad (10)$$

In the envelope region, where $\rho \to 0$ ($x \to \infty$) the assymptotic value of the monotonously increasing function $f(x)$ is equal to 1/4. This is realised on the surface with zero boundary condition for the gas pressure. Since the values of $x(r)$ are finite and positive throughout the stellar interior, for $x(R) \equiv x_R$ there applies $f(x_R) = \alpha/4$, $\alpha < 1$. On the other hand, $\eta(r)$ decreases with r increasing in the envelope to attain its minimum value $\eta(R) = 1$ on the surface. In view of this, relation (9) on the stellar surface it becomes

$$\kappa(R) < \alpha\frac{4\pi cG\mathcal{M}}{L} \,, \quad \alpha < 1 \,. \quad (11)$$

In the case $\alpha = 1$ this result is identical to the limitation for κ in the "standard model" (Eddington, 1926).

The dominant mechanism of absorption radiation in the envelopes of massive stars is Tomson's scattering on free electrons (opacity $\kappa_{\rm T}$). If there is in the envelope a value $x = x_0$ for which

$$f(x_0) = \frac{\kappa_{\rm T}}{C_*} \,, \quad (12)$$

then on the basis of (10) and (9) with $\kappa_{\rm T} \le \kappa(R)$ it follows $x_R > x_0$. In this case $T(x,\rho)$ from (2) for the photospheric values of the density ($\rho_{\rm ph}$) and temperature (T_e) yields

$$\rho_{\rm ph} < K_1^{-3}\frac{{T_e}^3}{x_0} \,. \quad (13)$$

The most favourable conditions for applying approximation (12) are in the vicinity of the inner boundary of the envelope. At this boundary (towards convective core) $\nabla_{\rm rad} = \nabla_{\rm ad}$, i. e.

$$\eta_k = C_*\frac{f(x_k)}{\kappa_k} \quad (14)$$

where the subscript 'k' denotes the values of the variables at the boundary. Throughout the stellar interior in the hydrostatic and thermal equilibria

$dP/dr < 0$ and the same is true also for the temperature and density gradients (unless in the latter case a homogeneous structure is considered). Therefore, in view of (4) within the convective core it there is in force

$$x_c > x(r) \geq x_k \ , \quad 0 < r \leq r_k \ , \qquad (15)$$

where x_c denotes the value of x at the centre. This relation, together with $x_R > x_0$, indicates that $(P_{\rm rad}/P_{\rm gas})$ has a minimum in the interior of the stars considered. The value of x_c can be assessed in the following way. By integrating (5) from the centre towards the surface, with $d\bar{\rho}(r)/dr \leq 0$ and $P(R) << P_c$, one obtains

$$P_c \leq \frac{3G}{8\pi}\left(\frac{\rho_c}{\bar{\rho}}\right)^{4/3}\frac{\mathcal{M}^2}{R^4} \ , \qquad (16)$$

where $\bar{\rho} \equiv \bar{\rho}(R)$. With $P_c(x,\rho)$ from (2), (16) becomes

$$x_c(1+x_c)^3 - A\mu_c^4 m^2 \leq 0 \ , \qquad (17)$$

with

$$m = \frac{\mathcal{M}}{\mathcal{M}_\odot} \ , \quad A = \frac{\pi a G^3}{18\mathcal{R}_g^4}\mathcal{M}_\odot^2 = 3.248\times10^{-2} . \ (18)$$

This result can be also obtained as a consequence of Theorem 7 (Chandrasekhar, 1939). Now, in view of (15) for $f(x)$, from (10) it holds

$$f(x_k) < f(x_c) \leq f(x_c^m) \ ,$$

where $x_c^m \equiv x_c^{max}$ is a real root of eq. (17). In this way from (12) and (14) with $\kappa_k \approx \kappa_{\rm T}$ one obtains

$$1 < \eta_k < \frac{f(x_c^m)}{f(x_0)} \ , \qquad (19)$$

since $\eta^{min} = \eta(R) = 1$. From here, for $l_k \approx 1$, it follows directly

$$\frac{f(x_0)}{f(x_c^m)} < q_k < 1 \qquad (20)$$

where $f(x_0) < f(x_c^m)$.

3. EVALUATION OF $\rho_{\rm ph}$, $\bar{\varepsilon}_{\rm core}$, $\mathcal{M}_{\rm core}$

The domains of possible values for $\rho_{\rm ph}$, η_k and q_k will be estimated from (13), (19) and (20), respectively. For this purpose a list of the observed quantities containing $\mathcal{M}$, L and T_e for the components of various double-star types belonging to the main sequence (Popper, 1980) will be used. In view of the approximation $\kappa \approx \kappa_{\rm T}$, only the upper branch of MS for Population I (with $\mu = 0.62$, $\kappa = 0.36$) will be considered.

Equation (12) in x_0 with $f(x_0)$ from (10) and $\gamma = 5/3$ reads

$$(C-4){x_0}^2 + \frac{1}{4}(C-20)x_0 - \frac{5}{8} = 0 \ , \qquad (21)$$

where

$$C = K\,\frac{\mathcal{M}/\mathcal{M}_\odot}{L/L_\odot} \ , \quad K = \frac{16\pi c G \mathcal{M}_\odot}{\kappa_{\rm T} L_\odot} = \frac{5.228}{\kappa_{\rm T}}\times10^4$$

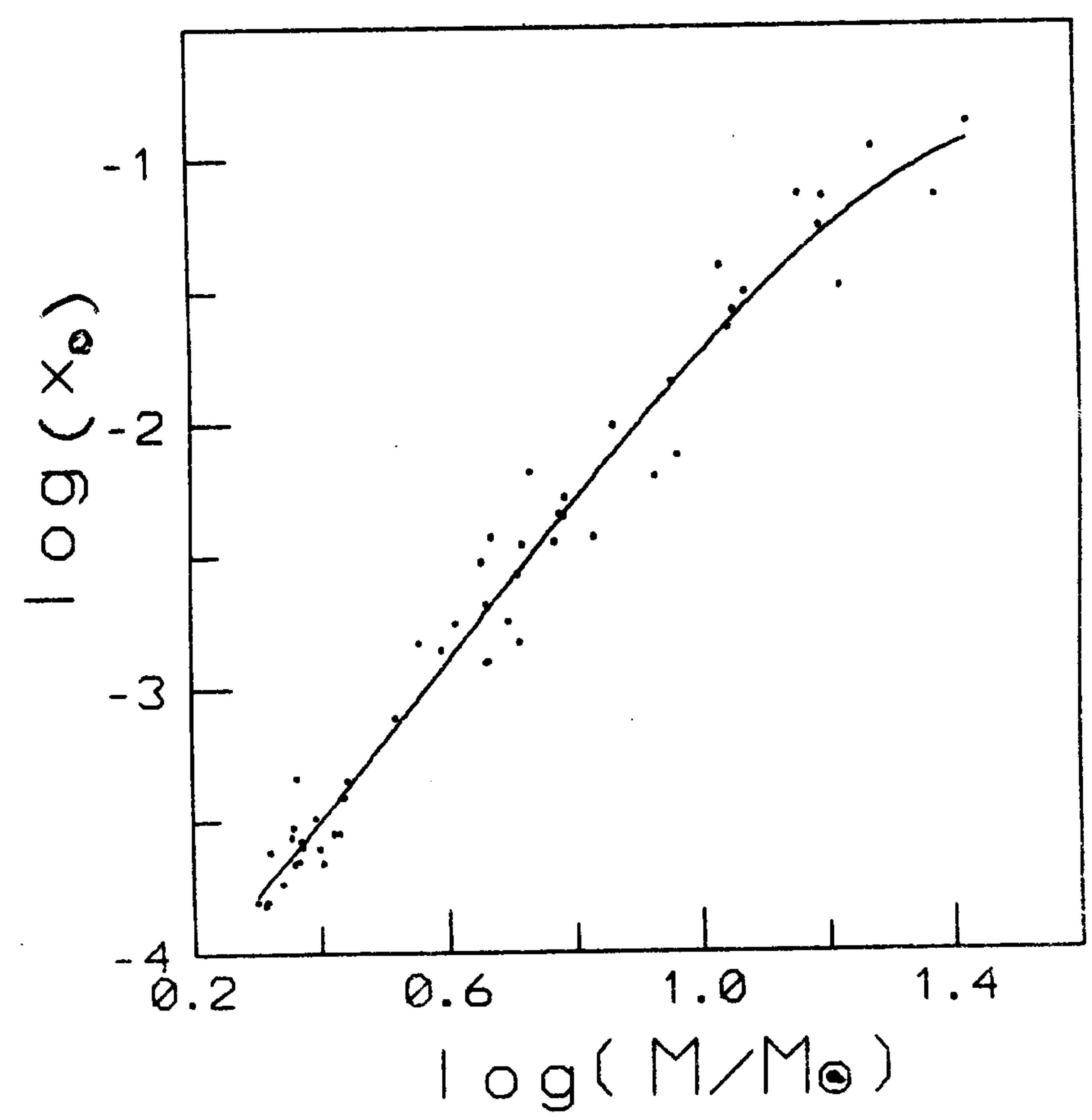

Fig. 1 The roots of equation (21)
- - fit to the mass-luminosity relation
— approximation (22)

and it has real and positive roots $x_0(\mathcal{M}, L)$. The correlation $x_0(\mathcal{M})$ can be derived by using the mass–luminosity relation for the considered region of MS (or for the entire main sequence, Angelov (1993)) or through the direct approximation in Fig. 1. In the latter case

$$\lg x_0 = -4.464 + 1.732z + 2.097z^2 - 1.107z^3 \ , \ (22)$$

with $z \equiv \lg(\mathcal{M}/\mathcal{M}_\odot)$ from (0.3, 1.43). For the measured T_e values the right-hand side of (13) can be considered in correlation with $\mathcal{M}$, i. e. L or M_b. In the latter case the line from Fig. 2 determines the upper limiting value $\rho^m_{\rm ph}(M_b)$ which cannot be attained by the photospheric density. After approximating the demarcation-line position, condition (13) becomes

$$\lg \rho_{\rm ph} < -7.0244 + 0.1152 M_b \ . \qquad (23)$$

Now condition (19) will be considered, where x_c^m (Fig. 3) should be determined. For $\mu_c = 0.62$ it is valid

$$x_c^m = 0.0143 - 0.0529z + 0.0768z^2 - 0.2067z^3$$

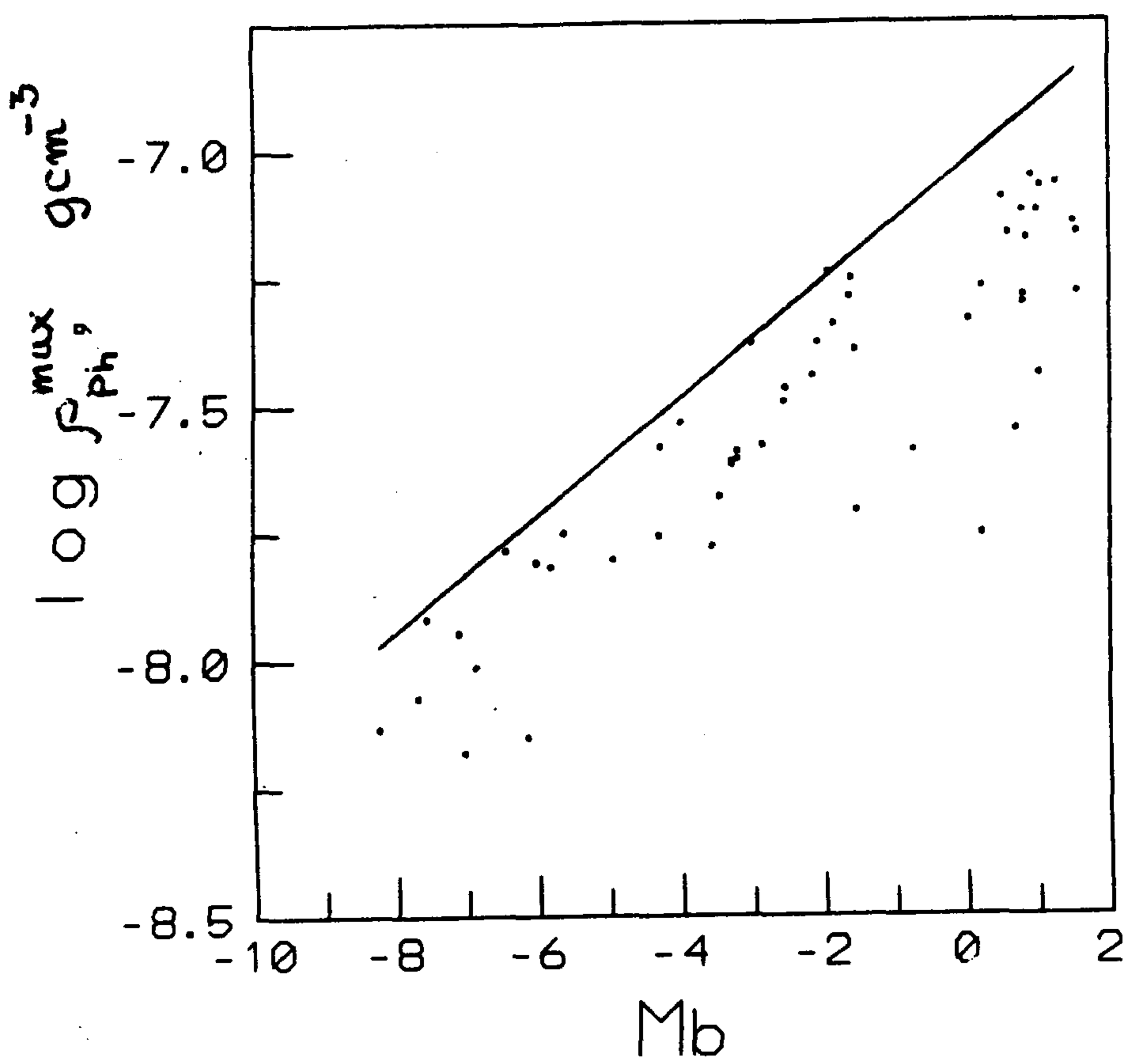

Fig. 2 The ρ_{ph}^{max} domain with uncorrelated x_0 from Fig.1 (details in the text).

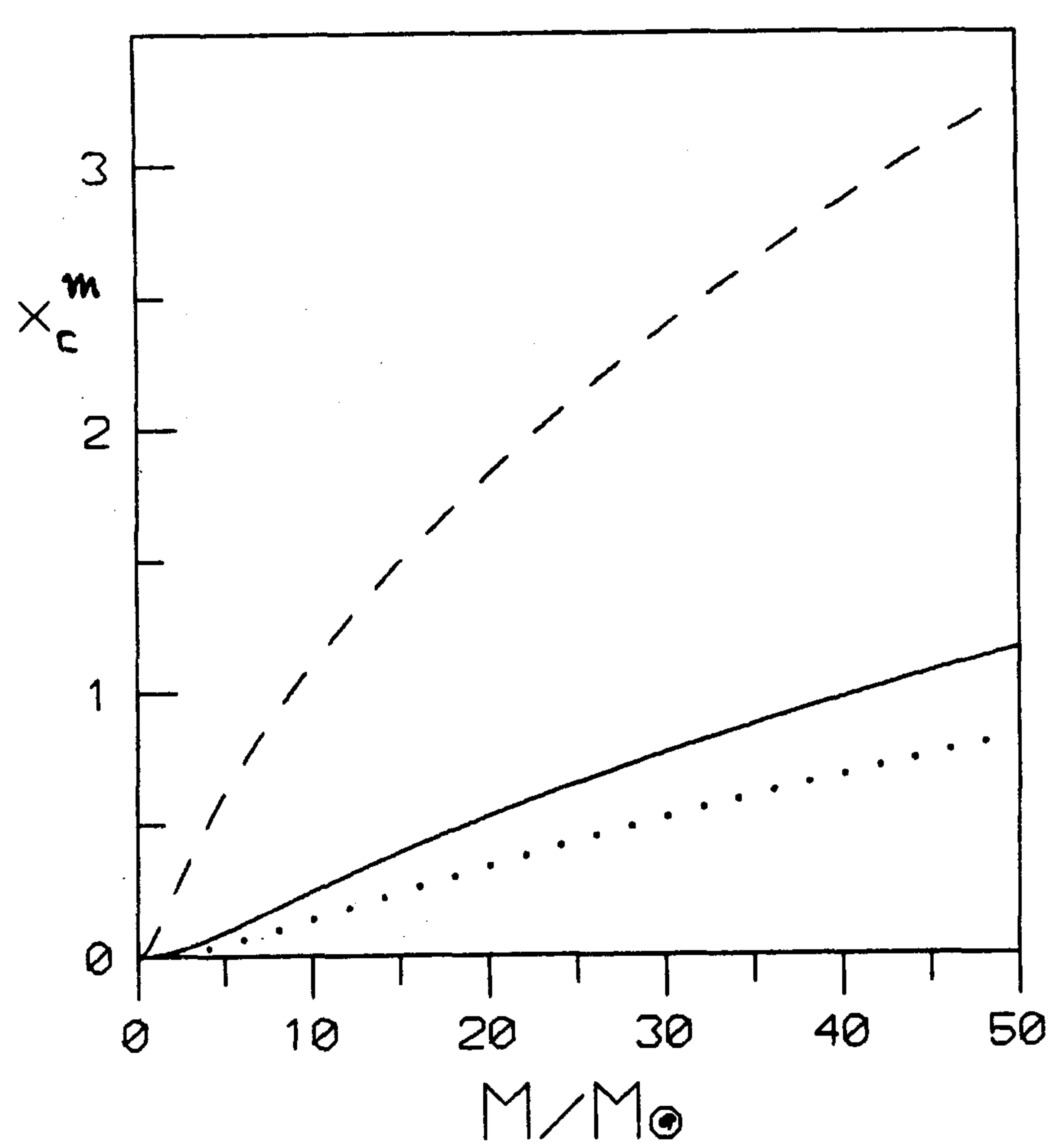

Fig. 3 The values for $x_c^m(\mathcal{M})$ according to (17) corresponding to total ionisation
... pure hydrogen ($\mu_c = 1/2$)
— Population I ($\mu_c = 0.62$)
- - pure helium ($\mu_c = 4/3$)

and one can also calculate $f(x_c^m)$ depending on the star's mass. The result is presented in Fig. 4 together with $f(x_0)$ corresponding to $x_0(\mathcal{M})$ according

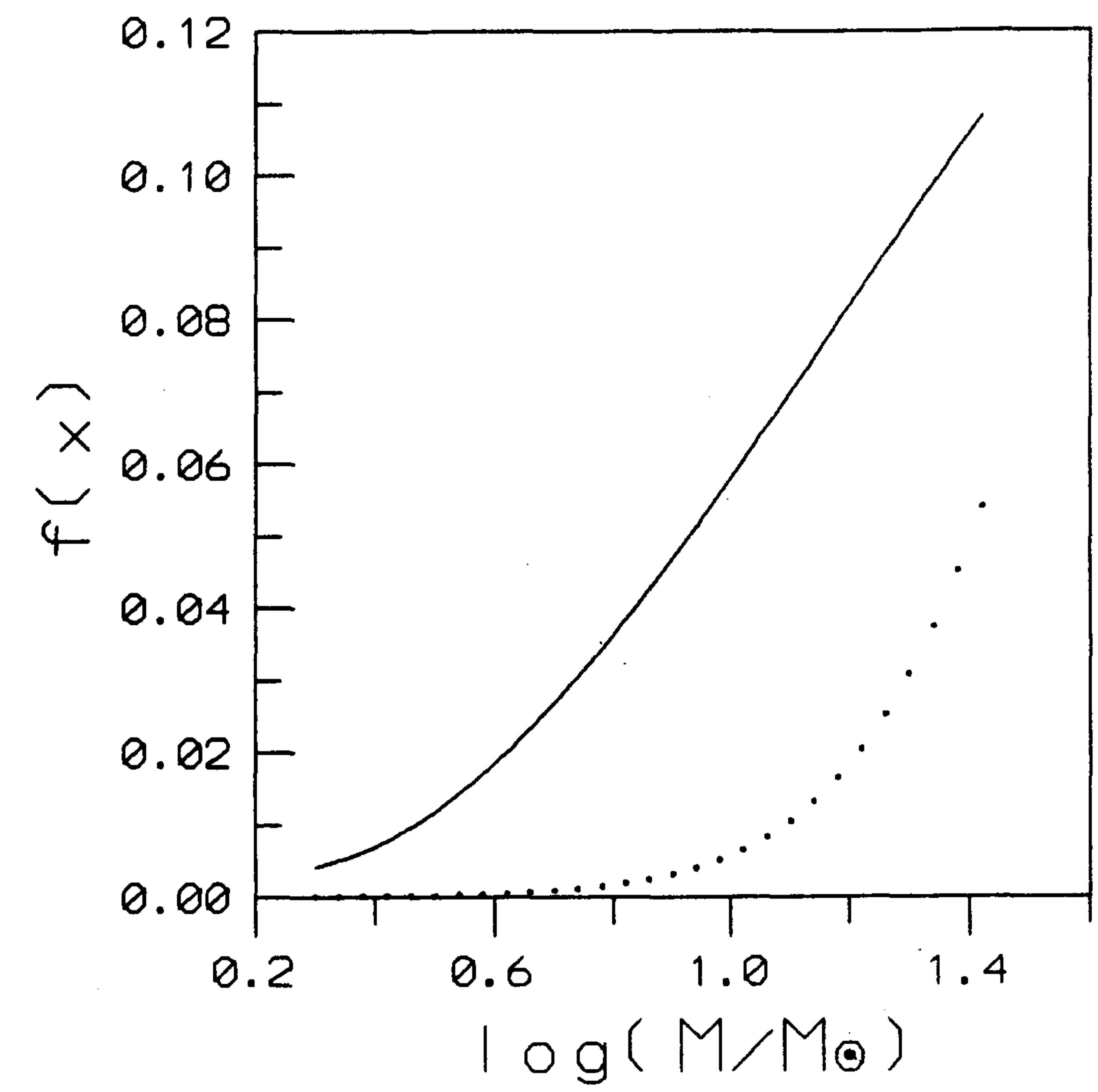

Fig. 4 The values for $f(x)$ from (10) for the upper branch of empirical MS
- - $x \equiv x_0$ ($\kappa_T = 0.36$)
— $x \equiv x_c^m$ ($\mu_c = 0.62$)

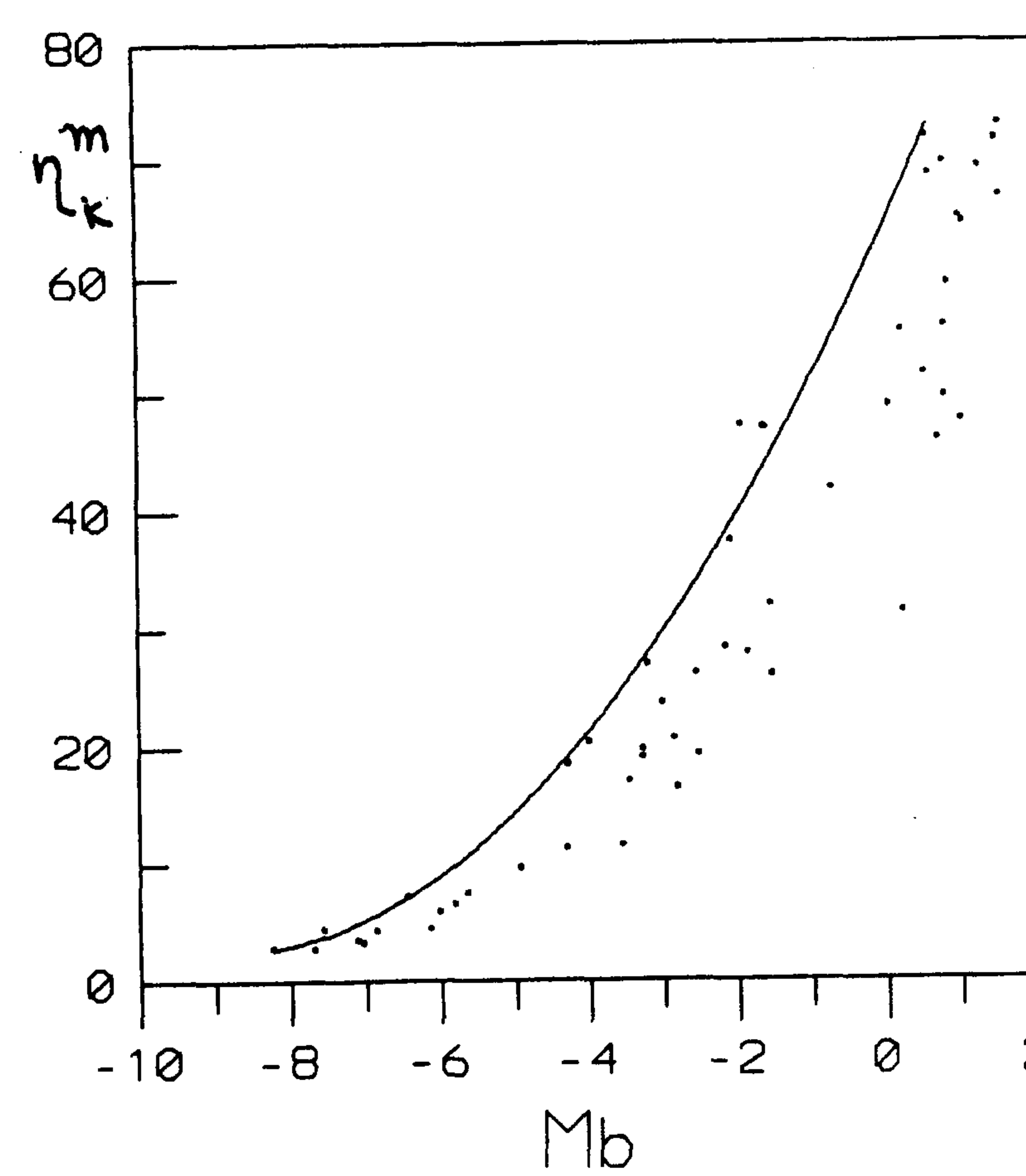

Fig. 5 The dependence $f(x_c^m)/f(x_0)$ on M_b for uncorrelated x_0 from Fig. 1. The polynomial in (24) approximates the demarcation-line location.

to (22). Finally, the correlations with M_b in (19) and (20) can be derived by using the *mass–luminosity* relation. Of course, a higher correctness is achieved if $f(x_c^m)/f(x_0)$ is calculated with uncorrelated x_0 from Fig. 1 and the limiting values are approximated by a polynomial in measured M_b. In this case for η_k (Fig. 5) and q_k (Fig. 6) one can write

$$\eta_k < 64.54 + 13.52 M_b + 0.73 M_b^2 \quad (\eta_k > 1) \quad (24)$$

and

$$\lg q_k > -1.82 - 0.08 M_b + 0.01 M_b^2 \quad (q_k < 1)\,, \quad (25)$$

for $-8^m40 \leq M_b \leq +1^m60$ on the main sequence.

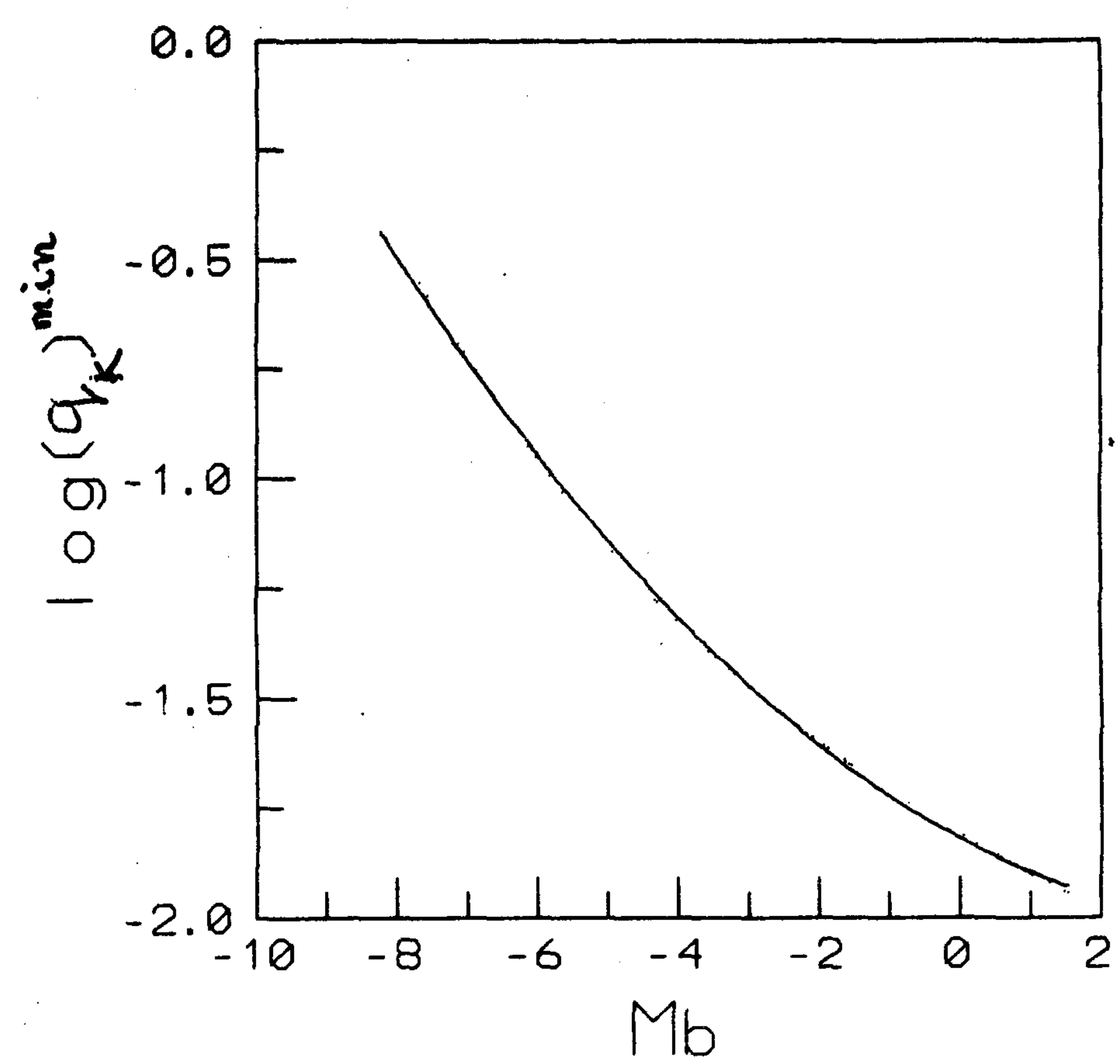

Fig. 6 The lower limit of the domain of values expected for log q_k for massive MS stars. Its location is approximated by the polynomial in (25).

In view of the definitions of η and q in (7) the criteria (19) and (20) can be written as

$$\frac{L}{\mathcal{M}} < \bar{\varepsilon}_{\rm core} < \eta_k^m \frac{L}{\mathcal{M}} \quad (26)$$

and

$$q_k^{min} \frac{\mathcal{M}}{\mathcal{M}_\odot} < \frac{\mathcal{M}_{\rm core}}{\mathcal{M}_\odot} < \frac{\mathcal{M}}{\mathcal{M}_\odot}\,, \quad (27)$$

respectively. With the values measured for L and $\mathcal{M}$ the domains (26) and (27) are presented in Figs. 7-8, whereas the corresponding correlations are

$$\lg \bar{\varepsilon}_{\rm core} \begin{cases} > 1.669 - 0.286 M_b \\ < 3.48 - 0.216 M_b - 0.0113 M_b^2 \end{cases} \quad (28)$$

and

$$\lg \frac{\mathcal{M}_{\rm core}}{\mathcal{M}_\odot} \begin{cases} > -1.34 - 0.187 M_b + 0.0116 M_b^2 \\ < 0.488 - 0.116 M_b\,. \end{cases} \quad (29)$$

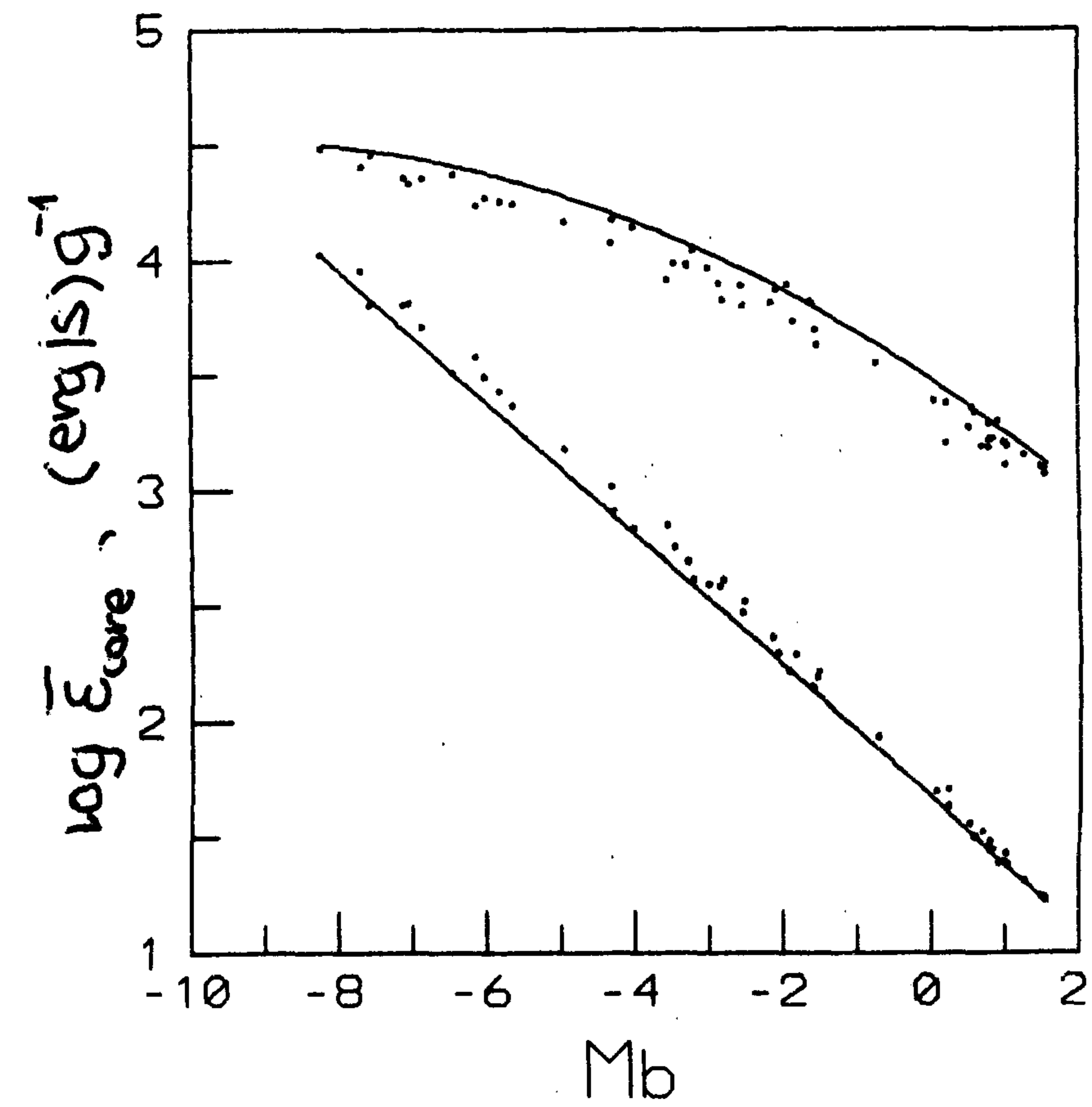

Fig. 7 The domain of values $\bar{\varepsilon}_{\rm core}$ – correl. (28).

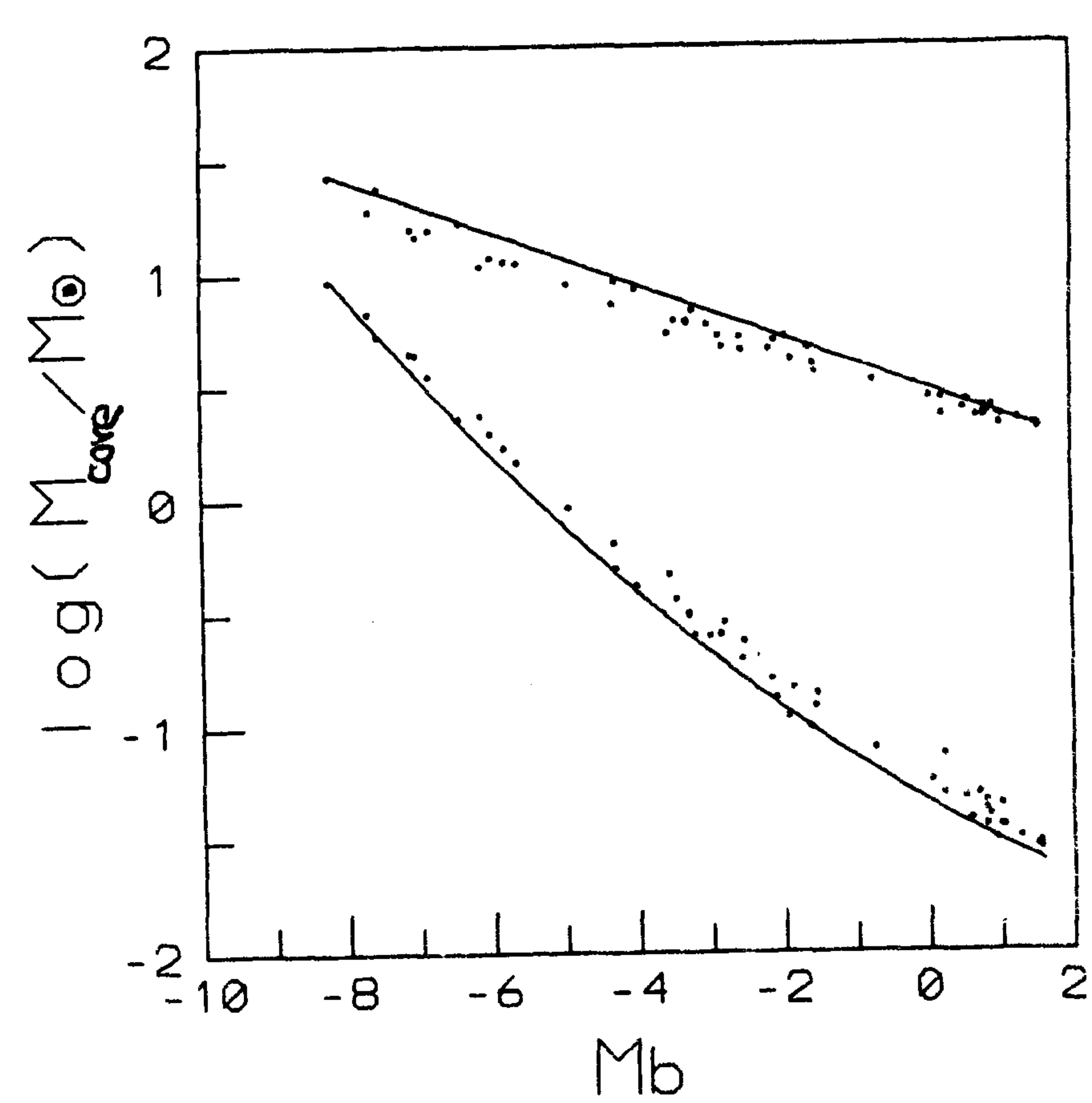

Fig. 8 The domain of values $\mathcal{M}_{\rm core}$ – correlation (29). The upper sequence of points is the diagram for the *mass–luminosity* relation(s) in the M_b interval considered.

4. CONCLUSIONS

With the measured values for $\mathcal{M}$, L and T_e for massive MS stars, the evaluation for $\rho_{\rm ph}$, $\bar{\varepsilon}_{\rm core}$ and $\mathcal{M}_{\rm core}$ has been done. Their interior is represented with a convective core in adiabatic equilibrium and with an envelope in the radiative equilibrium (with dominant Tomson's scattering). The starting relations are derived on the basis of a general criterion for the choice of the energy- transfer mechanism as function of the variable $(P_{\rm rad}/P_{\rm gas})$. This quantity has a minimum in the interior of the considered stars, whereas its maximum value at the centre can be estimated on the basis of the theorems concerning the global equilibrium of stars, also.

The correlations derived here are demarcation lines for the domains of expected values for $\rho_{\rm ph}$, $\bar{\varepsilon}_{\rm core}$ and $\mathcal{M}_{\rm core}$ in the case of stars of the upper MS branch in the interval $(-8^m40, +1^m60)$ for M_b. Globally considered, the photospheric density decreases with the increasing total mass of a star (for the MS region considered here, $\rho_{\rm ph}^{max} \approx 10^{-7} - 10^{-8}$ g/cm^3), whereas the values of $\bar{\varepsilon}_{\rm core}$ and $\mathcal{M}_{\rm core}$ (therefore also $L(r)$ at the boundary between the core and the envelope) increase progressively. In the same sense the criteria of estimating $\bar{\varepsilon}_{\rm core}$ and $\mathcal{M}_{\rm core}$ become more rigorous - the domains of values expected for these quantities become narrower as a consequence of decreasing deviation $\kappa_{\rm T}$ from κ. In this case the evaluation for $\bar{\varepsilon}_{\rm core}$ possesses a higher inner accuracy than the evaluation for $\mathcal{M}_{\rm core}$ for the same mass of the star. In any case the extrapolation of the results towards extremely massive stars ($M_b \leq -10^m$) is not desirable, due to the pulsation-instability phenomenon mainly.

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НЕКЕ КОРЕЛАЦИЈЕ ЗА МАСИВНЕ ЗВЕЗДЕ ГЛАВНОГ НИЗА

Т. Ангелов

Институт за астрономију, Студентски трг 16, 11000 Београд, Југославија

УДК 524.31
Оригинални научни рад

Изводе се критеријуми за процену вредности фотосферске густине, масе језгра и брзине генерације енергије масивних звезда главног низа. На основу посматрачког материјала за $\mathcal{M}, L$ и T_e, одређују се демаркационе линије области очекиваних вредности ових величина у корелацији са измереним M_b.