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Preliminary report

ON THE ORBITAL MOTION OF A BODY IN A SPHERICALLY SYMMETRIC FIELD IN ROSEN'S BIMETRIC GRAVITATION THEORY

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SUMMARY: The purpose of this note is to analyze some details concerning the possibility of accelerated orbital motion in the case of the one body problem in Rosen's bimetric gravitation theory.

Rosen's bimetric gravitation theory (one of two bimetric theories of the same author, both alternative to classical relativity) has been formulated in the first place as a theory which allows a generally covariant expression for gravitational energy, a quantity which has not been formulated in a satisfactory way in classical relativity. Further, the analysis of the spherically symmetric gravitational field showed that horizons, i.e. black holes of the Schwarzschild type, were not allowed in that theory. Moreover, the limit of stability of stellar masses was several times greater than in classical relativity [1]. But bimetric gravitation theory encountered serious difficulties in an attempt to explain the accelerated orbital motion of the binary pulsar 1913+16 [2]. An attempt was therefore made in [4] to establish the possibility of an accelerated orbital motion in a spherically symmetric field, based on a possibility of existence of <u>nonstatic</u> fields of that type, not allowed in classical relativity. It appears that such fields are allowed in Rosen's theory under relatively simple conditions.

We shall begin with a brief survey of relevant formulas for a static field.

a) We consider the spherically symmetric line element in Rosen's theory, as obtained in [1]

$$\sum ds^2 = e^{2M/r}(dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\lambda^2) - e^{-2M/r} dt^2 \tag{1.1}$$

The coordinates in (1.1) are isotropic; the Schwarzschild line element, written with respect to that system, reads

$$\sum ds^2 = (1 + \frac{M}{r})^4 (dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 \vartheta d\lambda^2) - (\frac{1 - M/2r}{1 + M/2r})^2 dt^2 \tag{1.2}$$

the transformation from purely radial coordinates R, with respect to which that element is habitually formulated, being

$$R = r(1 + M/2r)^2 \tag{1.3}$$

There results, from (1.1) and (1.2), that the metric coefficients of the bimetric field, when compared to those of the Schwarzschild solution, agree to the second order in M/r for the time-like part and to the first order for the space-like part. That is sufficient, as is pointed in [1], to bring about the same effects in the Solar System as those predicted by classical relativity; the accuracy of instruments of the present time is not sufficient to observe higher order effects, which could be relevant for verifying the validity of different theories.

It has been proven in [3], [4] that bimetric gravitation theory allows metrics conformally equivalent to (1.1), the conformal factor being the function of a solution of the wave equation in the Minkowskian metric, expressed with respect to spherically symmetric coordinates

$$\varphi_{|\epsilon\underline{\epsilon}} = 0 \qquad (1.4)$$

where the bar denotes covariant differentiation with respect to the Minkowskian metric and underlined indices mean that they are raised with the help of corresponding metrical coefficients. We assume φ isotropic, i.e. dependent on the radial coordinate r and on coordinate time t only. Spherically symmetric $d\tilde{s}^2$, conformally equivalent to (1.1), then reads

$$d\tilde{s}^2 = e^{2\varphi} ds^2 \qquad (1.5)$$

This metric, at the difference from (1.1), is no more static. This makes a notable difference from classical relativity, where such solutions, by Birkhoff's theorem, are not possible.

b) The differential equations of time-like geodesics in the equatorial plane of the central body determine, as usual in relativistic theories, the orbits of planets in the gravitational field considered. In the metric (1.5) we have first the Keplerian integral

$$r^2 e^{2[Mr^{-1}+\varphi(t,r)]}\frac{d\lambda}{d\tilde{s}} = l = \text{const} \qquad (1.6)$$

Then, substituting by (1.6), proper time $\tilde{s}$ by the angular coordinate λ in the remaining two equations of motion, we obtain

$$r'' - \frac{2}{r}(1-\frac{M}{r})r'^2 - (r-2M) +$$

$$+ l^{-2}r^4 e^{2(Mr^{-1}+\varphi)}(\frac{\partial\varphi}{\partial r} + \frac{M}{r^2}) = 0 \qquad (1.7)$$

$$t'' - \frac{2}{r}(1-\frac{2M}{r})r't' - l^{-2}r^4 e^{2(3Mr^{-1}+\varphi)}\frac{\partial\varphi}{\partial t} = 0 \qquad (1.8)$$

As a result [4] the orbital motion of the test body has an angular acceleration if, and only if

$$\frac{\partial\varphi}{\partial t} < 0 \qquad (1.9)$$

If we assume, similarly, φ acting in the sense of decreasing the radius vector in successive orbits, we have the inequality

$$e^{2\varphi}(\frac{\partial\varphi}{\partial r} + \frac{M}{r^2}) > \frac{M}{r^2} \qquad (1.10)$$

c) We shall consider now the solution φ (1.4) corresponding to the case of an outgoing wave, given [5] by

$$\varphi = \frac{1}{r}\psi(t-r) \qquad (1.11)$$

By differentiating one obtains directly from the above relation

$$\frac{\partial\varphi}{\partial r} + \frac{\partial\varphi}{\partial t} = -\frac{1}{r}\varphi \qquad (1.12)$$

ψ being an arbitrary function of $t-r$.

One distinguishes two possibilities:

1) $\varphi > 0$. Then for positive $\frac{\partial\varphi}{\partial r}$ inequality (1.10) becomes trivial. For negative $\frac{\partial\varphi}{\partial r}$ one has

$$|\frac{\partial\varphi}{\partial r}| < \frac{M}{r^2} \qquad (1.13)$$

2) $\varphi < 0$. Then by (1.9) and (1.12) one has

$$\frac{\partial\varphi}{\partial r} > |\frac{\partial\varphi}{\partial t}| \qquad (1.14)$$

This case is interesting in the sense that it acts "against" the metric (1.1) ($\varphi < 0, 2M/r > 0$).

d) Let us assume, by a procedure usual in relativity, the gravitational field sufficiently weak to allow a linear approximation. We shall obtain the decrease of mutually corresponding radial distances in succesive revolutions [4] for

$$\frac{\partial\varphi}{\partial r} > 0 \qquad (1.15)$$

Now, assuming only (1.9) satisfied, there appears by (1.12) that $\varphi < 0$ is a sufficient condition for (1.15) and (1.14). Conversely, if one assumes only (1.15) satisfied, with the complementary condition $\varphi > 0$, one obtains (1.9), with the additional inequality $|\partial_t\varphi| > \partial_r\varphi$.

Finally, on account of (1.9) and (1.15), but with no particular condition on φ, one obtains from (1.12)

$$\frac{\partial\varphi}{\partial r} > -\frac{1}{r}\varphi > \frac{\partial\varphi}{\partial t} \qquad (1.16)$$

e) Consider the general solution of the wave equation (1.4) [5], expressed as a combination of ingoing and outgoing waves

$$\varphi = \frac{1}{r}[\psi_1(t-r) + \psi_2(t+r)] \qquad (1.17)$$

Conditions (1.9) and (1.15) then yield

$$-\varphi - \frac{\partial \psi_1}{\partial (t-r)} + \frac{\partial \psi_2}{\partial (t+r)} > 0 \tag{1.18}$$

$$\frac{\partial \psi_1}{\partial (t-r)} + \frac{\partial \psi_2}{\partial (t+r)} < 0 \tag{1.19}$$

wherefrom

$$\frac{\partial \psi_1}{\partial t} < -\frac{1}{2}\varphi \tag{1.20}$$

It is interesting that the derivatives with respect to time of the function corresponding to the outgoing wave is the only to be limited by the magnitude of φ; (1.14) represents a "half" of the inequality (1.16).

We note that the inequality (1.15) is in general correct, and that (1.10) has to be taken instead only in the case when the distance between the central body and the satellite is very small.

The essential result obtained in [4] shows that accelerated orbital motion is possible in Rosen's theory, under conditions which appear to be simple, at least for the one body problem. It is a consequence of the fact that Birkhoff's theorem from classical relativity, by which a spherically symmetric gravitational field cannot be nonstatic (or time dependent), does not hold in Rosen's theory. In other words, monopole gravitational radiation (by Synge's definition [6]) is possible. The theoretical possibility of existence of nonstatic fields (1.5), conformally equivalent to the basic static field (1.1) is quite simple and has nothing artificial in itself; after all, there are in classical relativity solutions of the gravitational field equations which are purely radiative, like the Einstein-Rosen cylindrical metrics [6]. Of course, the question of the physical background of conformal nonstatic fields remains open. Perhaps such fields exist in every spherically symmetric solution, but the phenomenon is too small to be observed in stable systems like the Solar System and would be observable under some conditions of instability? In any case, the inequalities obtained in this paper show that the variety of solutions allowed is very large, the restrictions implied by them not being strong. We have restricted ourselves to the qualitative aspect of the questions considered, since an attempt at making quantitative assumptions would be too arbitrary.

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О КРЕТАЊУ ПО ОРБИТИ ТЕЛА У СФЕРНО СИМЕТРИЧНОМ ПОЉУ У ROSEN-ОВОЈ БИМЕТРИЧКОЈ ТЕОРИЈИ ГРАВИТАЦИЈЕ

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Претходно саопштење

У овом се раду испитују услови под којима долази до убрзаног обилажења небеског тела око гравитационог извора у Rosen-овој биметричкој теорији гравитације. Полази се од претпоставке да се небеско тело креће сразмерно близу гравитационог извора (знатно већег небеског тела), али је ипак довољно удаљено да би се у основним формулама могла извршити линеаризација. Поремећајни фактор који зависи од времена (што је начелно немогуће у класичној релативности) допушта убрзано обилажење и представља се прво као емисионо, затим као емисионо-апсорпционо решење таласне једначине.