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ON THE DYNAMICAL MASSES OF VISUAL BINARIES

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SUMMARY: The equations for the calculation of the dynamical masses of the visual–binaries components are derived. The method utilises the nonlocal and nonlinear empirical mass–luminosity relation which enables its application along the entire main sequence. The testing gives very good results.

1. INTRODUCTION

The mass–luminosity relation can be used for the purpose of calculating the parallaxes and masses of visual binaries (Russel, Moore, 1940; Baize, Romani, 1946). For the given orbit of a system and m_b of its components, the method accuracy depends on the choice of a particular log L – log $\mathcal{M}$ relation, i. e. log $\mathcal{M}$ – M_b. On the basis of observations, linear relations of this type have been derived for different regions of the main sequence: Harris *et al.* (1963) for $M_b \in [0^m, 7^m5]$ and $M_b \in [7^m5, 11^m]$; McCluskey, Condo (1972) for $M_b \in [-8^m, 10^m5]$; de Jager (1980) for $M_b \in (-12^m, -7^m)$.The coefficients of the mass–luminosity relation depend on the star's total mass, the chemical composition through its interior, the law of energy release and on the transfer mechanisms. Therefore, an increased number of linear log L(log $\mathcal{M}$) relations for covering the main sequence results in enlarging of their local accuracy, but diminishes the application domain for such a relation (analysis of this question is given in Angelov, 1993). In principle, the method of calculating the dynamical masses by using a unique linear log $\mathcal{M}$ – M_b relation yields insufficiently accurate results if for the system components significantly different mass–luminosity relations are valid.

In this paper a nonlinear log $\mathcal{M}(M_b)$ approximation is used along the entire main sequence. In Section 2 the equations for calculating the dynamical parallaxes and masses of the visual–binaries components are derived. The results of method testing to the systems with known trigonometric parallax and measured mass ratio of the components are given in Section 3.

2. THE CALCULATION OF THE DYNAMICAL PARALLAXES AND MASSES

One will use Kepler's law

$$\mu_1 + \mu_2 = \frac{a^3}{P^2 p^3} , \qquad (1)$$

the empirical mass–luminosity relation in the form

$$\log \mu = \sum_0^n A_k M^k \tag{2}$$

and Pogson's equation

$$M = m_b + 5 + 5\log p \ . \tag{3}$$

Here p'', a'' and P^y are the parallax, the semimajor axis and the period of a double star, respectively; $M \equiv M_b$ and $m_b = m_v + \mathrm{B.C.}$ are the bolometric magnitudes of the components (absolute and apparent), $\mu = \mathcal{M}/\mathcal{M}_\odot$ is the mass of one of the system components in units of $\mathcal{M}_\odot$ (μ_1 and μ_2 are the masses of the A and B components, respectively). The A_k coefficients in (2) are determined from the fit $\log \mu - M_b$ along the main sequence (Fig. 1). For the measured values of a and P of the system whose components belong to the main sequence, equations (1)–(3) determines the dynamical parallax and the masses of the components.

By using (3) for the i–th system component ($i = 1, 2$) with

$$y_i = m_{bi} + 5 \ , \quad z = 5\log p \ , \tag{4}$$

relation (2) becomes

$$\log \mu_i = A_0 + S_i \tag{5}$$

where

$$S_i = \sum_{k=1}^n A_k \sum_{j=0}^k \binom{k}{j} y_i^{k-j} z^j \tag{6}$$

Also, equation (1) becomes

$$\mu_i D_i = \alpha \ 10^{-0.6z} \tag{7}$$

where

$$\alpha = \frac{a^3}{P^2} \ , \ D_i = 1 + 10^{-\epsilon \Delta S} \tag{8}$$

with

$$\Delta S = S_2 - S_1 = \log \frac{\mu_2}{\mu_1} \tag{9a}$$

based on (5) and

$$\epsilon = \begin{pmatrix} -1, & i=1 \\ +1, & i=2 \end{pmatrix} . \tag{9b}$$

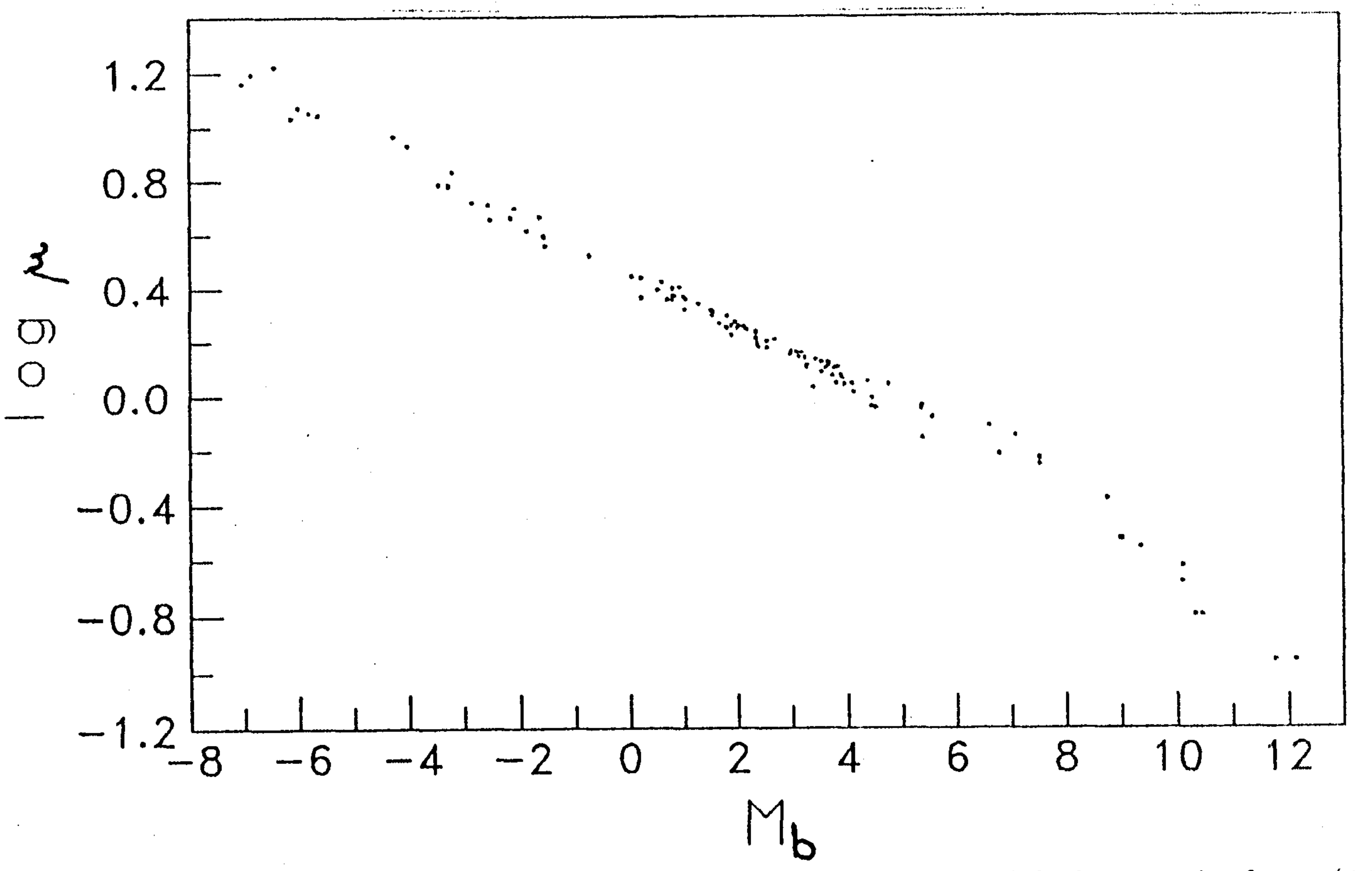

Fig. 1. The mass–luminosity diagram for the main sequence with Popper's data (1980).

Finally, for $\mu_2 \sim \mu_1$, by expansion

$$\log D_i \approx \log 2 - \frac{\epsilon}{2} \Delta S \tag{10}$$

equation (7) for any $i = 1, 2$ becomes

$$A_0 + \log \frac{2}{\alpha} + 0.6z + \frac{1}{2}(S_1 + S_2) = 0 \ ,$$

or in view of (6),

$$\sum_0^n E_j z^j = 0 \ . \tag{11}$$

The coefficients in (11) are

$$E_j(A_k, y_1, y_2, \alpha) = e_j + \frac{1}{2} \sum_{k=j}^n \binom{k}{j} A_k s_{k-j} \tag{12}$$

with

$$s_{k-j} = y_1^{k-j} + y_2^{k-j} \qquad (13a)$$

and

$$e_0 = \log \frac{2}{\alpha} \ , \quad e_1 = 0.6 \ , \quad e_{j>1} = 0 \ . \qquad (13b)$$

On the basis of (4) and $p < 1''$, the solution of (11) looked for here is the real value $z < 0$ and the dynamical parallax $p = 10^{0.2z}$.

With the solution for z from (11), equation (7) through (10) determines the component masses:

$$\log \mu_i = \log \frac{\alpha}{2} - 0.6z - \frac{\epsilon}{2} \triangle S \ .$$

If $\triangle S$ according to (9a) is expressed via S_i from (6), the upper equation becomes

$$\log \mu_i = \sum_0^n F_j z^j \ , \quad i = 1,2 \ . \qquad (14)$$

The coefficients in (14) are

$$F_j(A_k, y_1, y_2, \alpha, \epsilon) = -e_j + \frac{\epsilon}{2} \sum_{k=j}^{n} \binom{k}{j} A_k r_{k-j} \qquad (15)$$

with e_j from (13b), ϵ from (9b) and

$$r_{k-j} = y_2^{k-j} - y_1^{k-j} \ . \qquad (16)$$

3. TEST

The dynamical parallaxes p_d and the masses of the components μ_d of some visual binaries with measured p_{tr}, B.C. and $\mu_2/(\mu_1 + \mu_2)$ will be calculated. For the purpose of determining A_k in (12) and (15), the diagram presented in Fig. 1 will be approximated by a polynomial (2) with $n = 7$. One obtains

$$A_0 = +0.457591, \ A_1 = -0.104653,$$
$$A_2 = -0.144867 \times 10^{-2}, \ A_3 = +0.467526 \times 10^{-3},$$
$$A_4 = +0.153159 \times 10^{-3}, \ A_5 = -0.155916 \times 10^{-4},$$
$$A_6 = -0.217331 \times 10^{-5}, \ A_7 = +0.179296 \times 10^{-6} \ . \qquad (17)$$

Simultaneously the results for p_d and μ_d obtained by using two different linear relations $\log \mu - M_b$ will be presented. Namely, if in stead of (2) one uses

$$\log \mu = -k(M_b - M_\odot) \ , \quad k = const, \qquad (18)$$

the quantity D_i will be parallax independent. In this case expansion (10) is not necessary and from the initial equations one obtains

$$\frac{3-5k}{k} \log p = (5 - M_\odot) + m_{bi} - \frac{1}{k} \log \frac{D_i}{\alpha} \ ,$$
$$i = 1 \ or \ 2, \qquad (19)$$

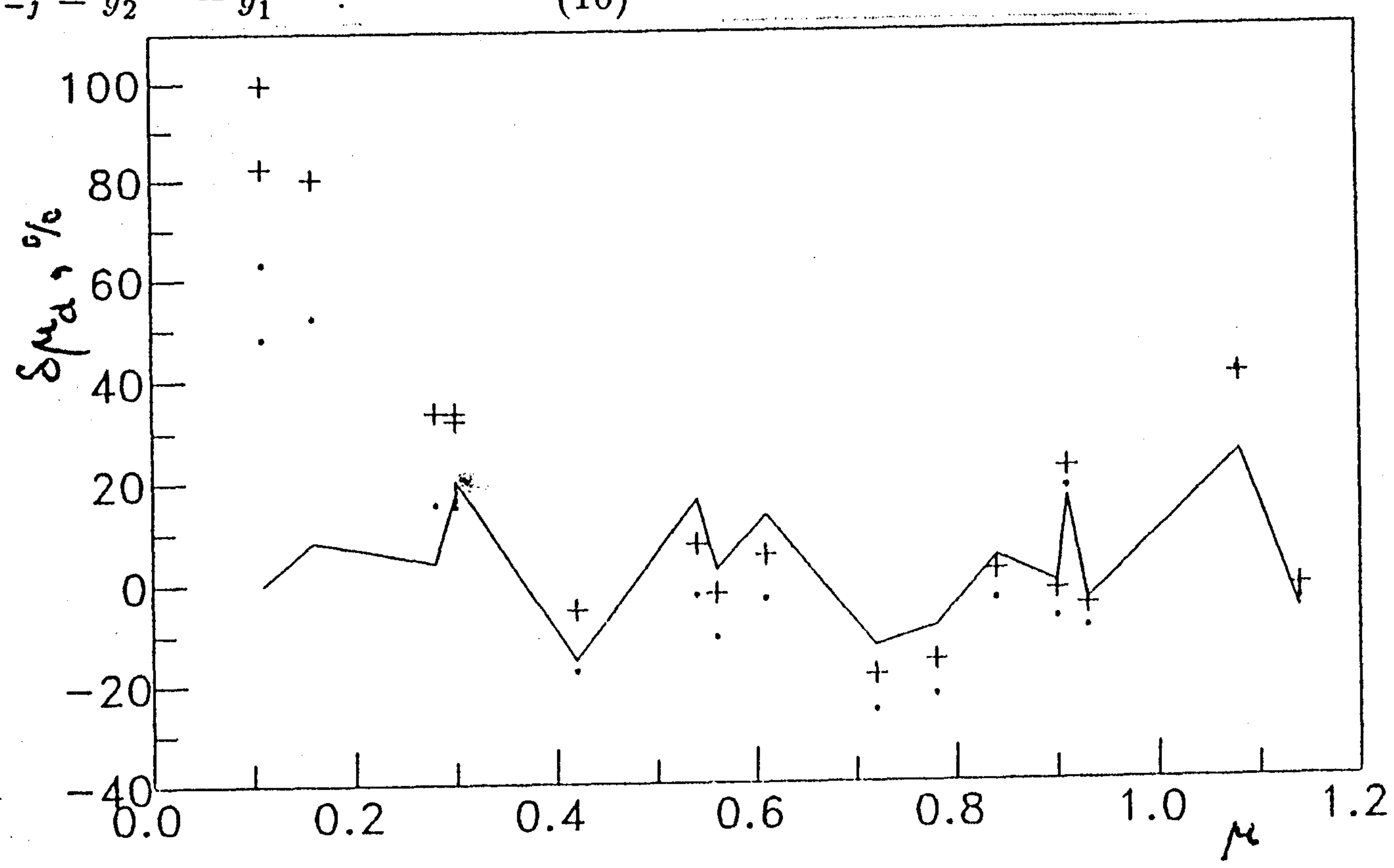

Fig. 2. The accuracy of formulas used for μ_d.
Ref. (see Table 1): $***$ (—) , $**$ (.) , $*$ (+).

Table 1. The results of testing

Binary	a''	P^y	m_b	p''_{tr}	p''_d			μ	μ_d		
					***	**	*		***	**	*
α Cen	17.56	79.9	0.04	0.743	0.758	0.760	0.751	1.14	1.07	1.09	1.12
HD128620/1	—	—	1.04	—		—		0.93	0.89	0.84	0.88
L726–8	2.06	26.5	8.84	0.385	0.385	0.331	0.309	0.11	0.11	0.18	0.22
Star B = UV Cet	—	—	9.21	—		—		0.11	0.11	0.16	0.20
Kr 60	2.38	44.4	7.35	0.250	0.249	0.229	0.218	0.28	0.29	0.32	0.37
$+56^0 2783$	—	—	8.45	—		—		0.16	0.17	0.24	0.29
70 Oph	4.55	88.1	4.03	0.203	0.198	0.206	0.201	0.84	0.88	0.81	0.86
HD165341	—	—	5.25	—		—		0.61	0.69	0.59	0.64
η Cas	11.99	480	3.36	0.172	0.169	0.168	0.165	0.91	1.05	1.07	1.11
HD4614	—	—	6.34	—		—		0.56	0.57	0.50	0.55
Wolf 630	0.22	1.742	7.71	0.161	0.170	0.172	0.164	0.42	0.36	0.35	0.40
HD152751	—	—	7.71	—		—		0.42	0.36	0.35	0.40
Fu 46	0.71	13.0	8.04	0.153	0.144	0.145	0.138	0.30	0.36	0.35	0.40
HD155876	—	—	8.09	—		—		0.30	0.35	0.35	0.40
ξ Boo	4.92	152	4.51	0.148	0.151	0.156	0.152	0.90	0.90	0.83	0.88
HD131156	—	—	6.24	—		—		0.72	0.63	0.53	0.59
HR6426	1.82	42.1	5.93	0.137	0.137	0.144	0.140	0.78	0.71	0.60	0.66
HD156384	—	—	6.46	—		—		0.54	0.63	0.53	0.58
γ Vir	3.75	171.4	3.52	0.094	0.087	0.084	0.084	1.08	1.35	1.52	1.51
HD110379/80	—	—	3.52	—		—		1.08	1.35	1.52	1.51

The systems with both components (A and B) on the main sequence are from Popper (1980). The references for p_d and μ_d are:

*** eqs. (11) and (14) with A_k from (17).

** eqs. (19) and (20), with $k = 0.1117$ and $M_{\odot} = 4^m 77$ from Baize, Romani (1946); see and Couteau (1978).

* eqs. (19) and (20), with $k = 0.103$ and $M_{\odot} = 4^m 89$ from McCluskey, Kondo (1972); see and Dommanget, Lampens (1993).

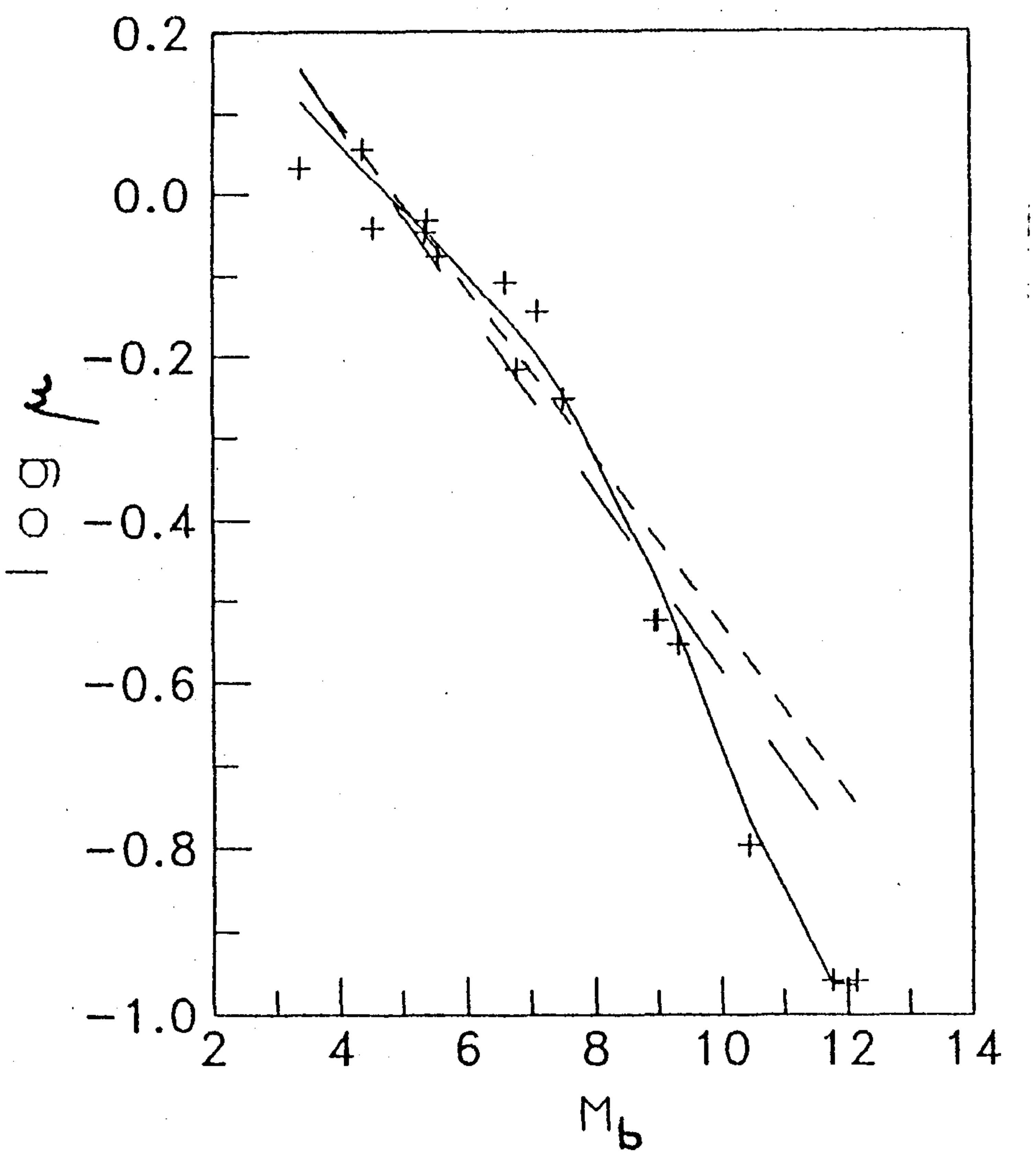

Fig. 3. The testing of the mass–luminosity relation in the domain of visual binaries considered.
Ref: $***$ (—) , $**$ (— —) , $*$ (- - -).

$$\frac{3-5k}{3k}\log\mu_i = -(5 - M_\odot) - m_{bi} + \frac{5}{3}\log\frac{D_i}{\alpha}\,,$$

$$i = 1, 2, \qquad (20)$$

with α, D_i from (8) and $\Delta S = -k(m_{b2} - m_{b1})$.

The diagram on Fig. 2 illustrates the accuracy of formulas applied ($\delta\mu_d$, in %, is relative deviation μ_d with respect to μ).The linear approximations (18) are practically equally suitable in $0.4 < \mu < 1$, although the nonlinear approximation (2) gives a better estimate ($\bar{\delta}\mu_{***} = -1\%$, $\bar{\delta}\mu_{**} = -11\%$, $\bar{\delta}\mu_{*} = -4\%$). With decreasing of the mass in $\mu < 0.3$, the "unique" linear relation (18) gives μ_d with an increasing error which cannot be tolerated near the right boundary of the main sequence (which is to be expected on account of Fig. 3) — the simplicity of (20) has here a too high cost. At the same time (14) still yields very good results.

Finally, the diagram on Fig. 3 can be approximated by two (or more) local relations of type (18). However, their application to the calculation of μ_d becomes complicated if for the components of a system different mass–luminosity relations are valid. In general, compared to the results following from (20), formula (14) yields a higher accuracy of dynamical masses and it can be applied with a greater certitude to the whole main sequence.

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О ДИНАМИЧКИМ МАСАМА ВИЗУЕЛНО ДВОЈНИХ ЗВЕЗДА

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Оригинални научни рад

Изводе се једначине за израчунавање динамичких маса компонената визуелно двојних звезда. Метод користи нелокалну и нелинеарну емпиријску релацију маса–сјај што омогућава његову примену на целом главном низу. Тестирање даје врло добре резултате.