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# THE CHARACTER OF THE INFLUENCE OF THERMAL PROCESSES ON THE STABILITY OF SPIRAL DENSITY WAVES

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**SUMMARY: A numerical illustration of some effects of thermal processes in the stability domain of spiral density waves in the galactic disc is given.**

## 1. INTRODUCTION

In the theory of spiral density waves (Lin, Shu, 1964) the gaseous component of the galactic disc has been considered in the adiabatic equilibrium. The fundaments of the thermal-instability theory (the volume forces absent) were given by Field(1965), whereas the effects of gravitation on the thermal instability has been considered by several authors (e.g. Marochnik, Suchkov, 1984). The role of thermal processes in the response of the galactic disc to a given perturbation in the gravitation field, as well as the disc stability, has been considered by Angelov(1992a, 1992b). In the latter paper the dispersion equation for the cases of small spiral perturbations was obtained

$$(\overline{\nu}^2 - \nu^2)(\overline{\nu} - ix_T) - ia = 0, \quad \overline{\nu} \neq 0. \qquad (1)$$

Here we have

$$a = x^2\frac{(\gamma-1)x_T + x_\sigma}{\gamma}, \quad x^2 = \frac{k^2c_s^2}{\kappa^2},$$

$$x_T = \frac{(\gamma-1)\mu\mathcal{L}_T}{\kappa\mathcal{R}_g}, \quad x_\sigma = \frac{(\gamma-1)\mu\sigma^0\mathcal{L}_\sigma}{\kappa T^0\mathcal{R}_g}, \qquad (2)$$

$$\nu^2 = 1 - |k|_* + \frac{Q^2}{4}k_*^2 > 0,$$

$$\frac{Q^2}{4} = \frac{k_c^2c_s^2}{\kappa^2}, \quad |k|_* = \frac{|k|}{k_c}, \quad k_c = \frac{\kappa^2}{2\pi G\sigma^0};$$

$\mathcal{L}_\sigma$ and $\mathcal{L}_T$ are the partial derivatives of the gas heating-cooling function $\mathcal{L}(\sigma, T)$ in $\sigma$ and $T$, respectively, $c_s$ is the adiabatic sound speed in the disc plane of symmetry, $k_c$ is the Jeans wave number and

$$\overline{\nu} = \frac{Re(\omega) - m\Omega}{\kappa} + i\frac{Im(\omega)}{\kappa} \qquad (3)$$

is the dimensionless frequency (other designations in $x^2, x_T, x_\sigma$ and $\overline{\nu}$ are usual, with $\gamma$ in the plane of the disc).

In this paper some details of the heating-and-cooling effect in the stability domain of spiral waves in the galactic disc will be analysed.

## 2. THE CHARACTER OF THERMAL-PROCESS INFLUENCE

The dispersion equation (1) comprises the short (S) and the long waves (L) of both types ($l$ and $t$). If $k \in Re$ and $\overline{\nu} = u + iw$, from the real and imaginary parts of (1) one can obtain two equations:

$$F_1(w, |k|_*) = 0 \quad and \quad F_2(u^2, |k|_*) = 0,$$

with base-state parameters $Q^2, x_T, x_\sigma$. By analysing $F_1 = 0$ one derives a criterion of disc stability for all waves (Angelov, 1992b):

$$(\gamma - 1)x_T + x_\sigma > 0, \quad Q^2 \geq \frac{1}{1 - x_T^2/3}, \qquad (4)$$

which with $\alpha = x_T/x_\sigma$ and $x_\sigma > 0$ can be rewritten as

$$\alpha > -\frac{1}{\gamma - 1}, \quad |x_T| \leq \left(3\frac{Q^2 - 1}{Q^2}\right)^{1/2}. \qquad (5)$$

The second inequality in (4), i.e. in (5), in the presence of thermal processes ($\mathcal{L} \neq 0$) is fulfilled for $Q^2 > 1$ i.e. $|x_T| < \sqrt{3}$.

The explicit form of the equations $F_{1,2} = 0$ in the unknown quantity $|l| = |\lambda|/\lambda_c = 1/|k|_*$, where $\lambda_c = 2\pi/k_c$ is the Jeans wavelength, will be given:

$$F_1(|l|) \equiv Al^2 - B|l| + C = 0 \qquad (6)$$

and

$$F_2(|l|) \equiv \sum_{k=0}^{6} \sum_i (-1)^{k+i} a_{ik}(Q^2, x_T^2) u^{2i} |l|^k = 0. \qquad (7)$$

With the parameters

$$w_0 = x_T \frac{1 + (\gamma - 1)\alpha}{2\gamma\alpha},$$

$$h = \frac{x_T^2}{3}, \quad f = \frac{Q^2}{4},$$

the coefficients $A, B, C$ in (6) and $a_{ik} \neq 0$ in (7) are:

$$A = w[(2w - x_T)^2 + 1],$$

$$B = w, \quad C = f(w - w_0), \qquad (8)$$

and

$$a_{00} = f^3; \quad a_{01} = 3f^2;$$

$$a_{02} = 3f\{1 + f(1 + 8\frac{3}{16}h)\}, \quad a_{12} = 9f^2;$$

$$a_{03} = 1 + \frac{3}{2}(4 + 17h)f, \quad a_{13} = 18f; \qquad (9)$$

$$a_{04} = 3\{1 + 2h + \frac{f}{2}(2 + 17h + 9h^2)\},$$

$$a_{14} = 9\{1 + 2f(1 - h)\}, \quad a_{24} = 24f;$$

$$a_{05} = 3(1+h)(1+3h), \quad a_{15} = 18(1-h), \quad a_{25} = 24;$$

$$a_{06} = (1 + 3h)^2, \quad a_{16} = 9(1 - h)^2,$$

$$a_{26} = 24(1 - h), \quad a_{36} = 16.$$

At first the solutions of (6) in the stability domain ($w > 0$), presented in Fig.1$a$, will be analysed.

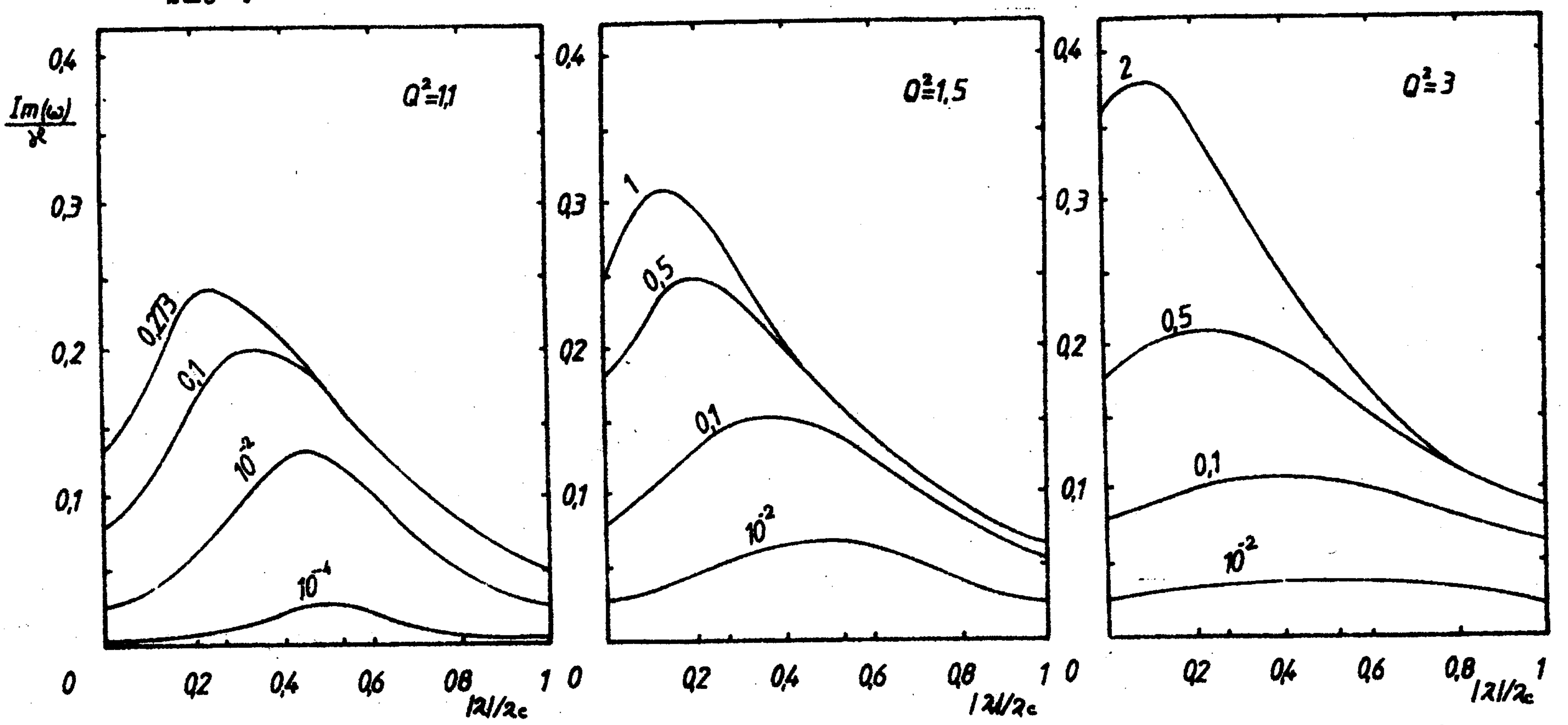


**Fig. 1$a$.** Solution of equation (6) in the stability domain (with $\alpha = -1/2$) for a few values of $x_T^2$, according to (5), with $Q^2 = 1.1, 1.5, 3$ given. The upper curve for each $Q^2$ corresponds to a value of $x_T^2(max)$.

For $C = 0$ the roots of the polynomial $F_1(|l|)$ yield $\lambda/\lambda_c = 0, \pm l_0$ with

$$l_0 = \frac{1}{x_T^2(\frac{1-\alpha}{\gamma\alpha})^2 + 1}.$$

For any $w_0 < w < w_m$ there are four types of solutions for the wavelength: two $t$-waves with $\lambda/\lambda_c$ from $(-l_0, 0)$ and two $l$-waves with $\lambda/\lambda_c$ from $(0, +l_0)$. In each pair of solutions there is one corresponding to the short(S) and one corresponding to the long waves(L). For $0 < w < w_0$ there is one solution for both $t-$ and $l-$waves with $|\lambda| > l_0\lambda_c$. According to Fig.1$a$ every $|x_T| \leq |x_T|_{max}$ generates one harmonic in both L- and S-modes for the $t$- and $l$-waves in the domain $w_0 < w < w_m$, and their number increases with $Q^2 > 1$ increasing. Also, the effect of thermal processes for a given $Q^2$ increases with $|x_T|$ increasing.

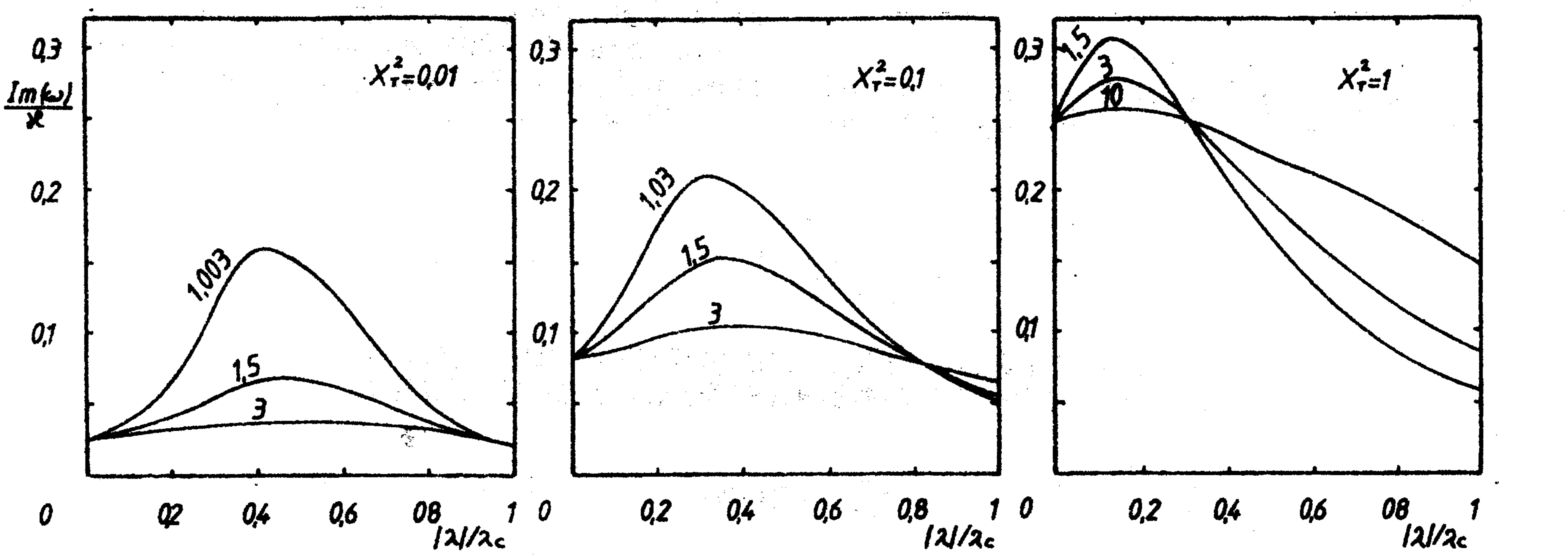


**Fig. 1$b$.** Solution of equation (6) in the stability domain (with $\alpha = -1/2$ ) for a few values of $Q^2$, according to (5), for each of the given values $x_T^2 = 0.01, 0.1, 1$.

For a given pair of values $Q^2, |x_T|$ from (5) the separation of the L- and S-modes of both wave types occurs at the maximum $(|l|_m, w_m)$ on the corresponding curve in Fig.1$a$. From the condition that $F_1(|l|)$ has one double root one obtains the equations for $w_m$ and $|l|_m$:

$$w_m - Q^2(w_m - w_0)[(2w_m - x_T)^2 + 1] = 0,$$

$$2w_m|l|_m - Q^2(w_m - w_0) = 0. \qquad (10)$$

Fig.1$b$ illustrates the second variant of the solution of equation (6), whereas Fig.2 presents the solution of system (10).

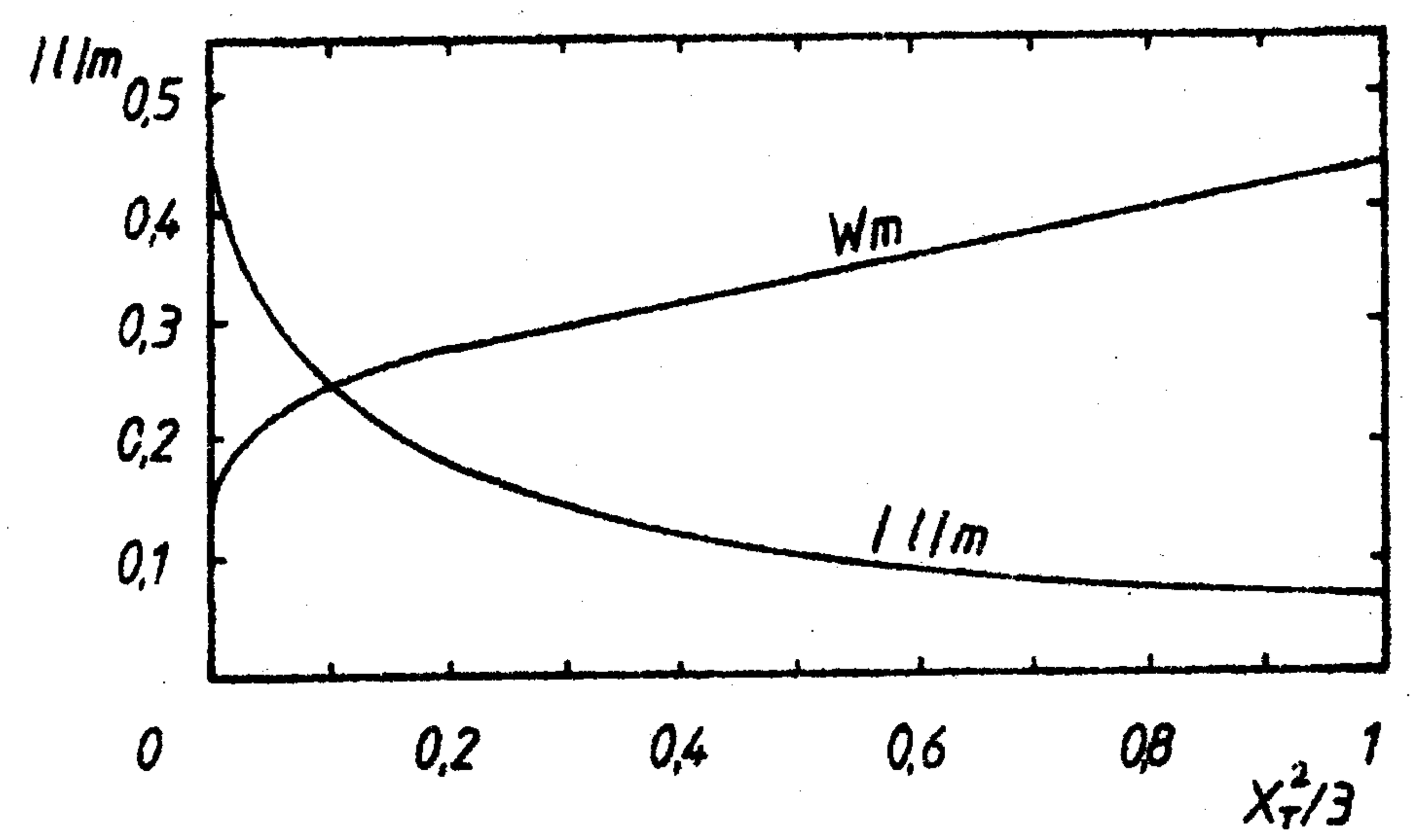


**Fig. 2.** Solution of system (10) depending on $|x_T|_{max}$ with $\alpha = -1/2$.

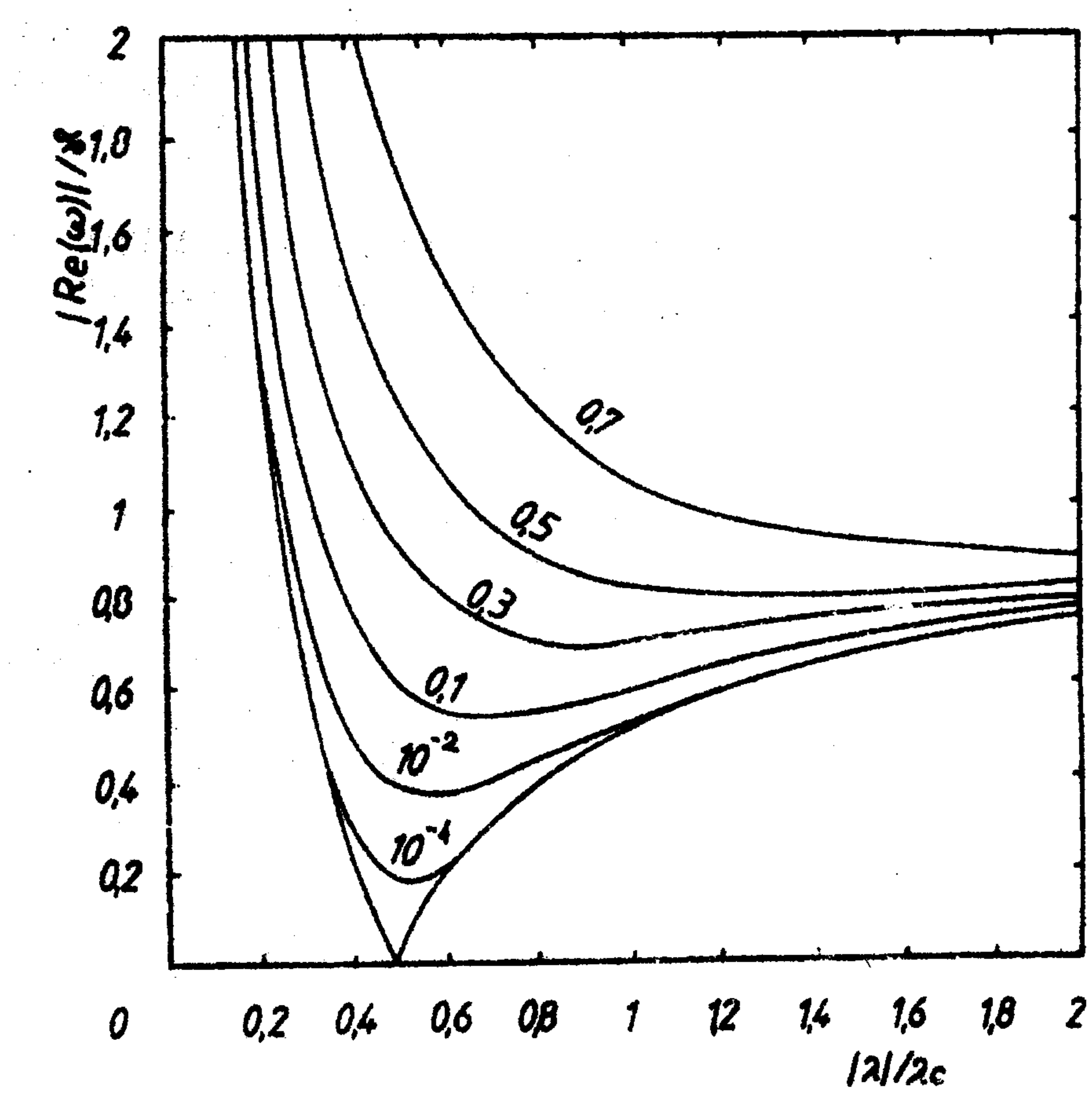


**Fig. 3.** Roots of polynomial (7) for a few values of $x_T^2(max)/3$.

The roots of polynomial (7) are presented in Fig.3. It is seen that the waves reach the corotation circle only in the absence of thermal processes ($x_T = 0$) when (7) is transformed in the well-known dispersion equation

$$(1-\nu^2)l^2-|l|+\frac{Q^2}{4}=0, \quad Q^2=1. \qquad (11)$$

As should be expected, the thermal processes enhance the stability (according to (5) when $|x_T|$ increases, $Q^2$ also increases). Already for $x_T^2 = 1.5$ ($Q^2 = 2$) one can hardly speak about any spiral structure.

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## КАРАКТЕР УТИЦАЈА ТОПЛОТНИХ ПРОЦЕСА НА СТАБИЛНОСТ СПИРАЛНИХ ТАЛАСА ГУСТИНЕ

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Даје се нумеричка илустрација неких ефеката топлотних процеса у област стабилности спиралних таласа густине у галактичком диску.