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THERMAL PROCESSES AND SPIRAL DENSITY WAVES I. RESPONSE OF THE GALACTIC DISC TO A PERTURBATION IN THE GRAVITATIONAL POTENTIAL

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SUMMARY: The gaseous disc of the Galaxy in differential rotation, with heating and cooling processes, is considered. The linear equations of small perturbations are derived by using the infinitesimally-thin-disc approximation. Solutions for the amplitudes of spiral perturbations in the density and velocities are given.

1. INTRODUCTION

The theory of spiral density waves in the galactic disc (Lin, Shu, 1964, 1966) gives solutions to two basic problems: 1. The character of the disc response to a given perturbation in the gravitational potential and 2. The evolution of small perturbations - analysis of the disc stability. Depending on the considered disc component the first problem in the classical "elementary" theory utilises the equations of gaseous dynamics for isothermal(or adiabatic) processes, i.e. the equations of stellar hydrodynamics. In the second problem the basic system is added by Poisson's equation - its asymptotic solution for the gravitational potential has been used in the derivation of the dispersion equation of small perturbations (e.g. Rohlfs, 1977; Marochnik, Suchkov, 1984).

The subject of the present paper is the solving of the first problem where the energy equation with a general heating-cooling function is also considered. The results are valid for the gaseous disc of the Galaxy, free of heat conduction, with no viscous and magnetic forces. In Section 2 the linear equations for small perturbations of an arbitrary type are derived by using the infinitesimally-thin-disc approximation. In Section 3 their solving for the amplitudes of spiral perturbations in the density and velocities for a given gravitational-potential perturbation is presented.

2. THE LINEAR EQUATIONS OF SMALL PERTURBATIONS

The fundamental differential equations for describing the dynamical state of a nonviscous gas in the gravitational field, with no magnetic forces and heat conduction, are

$$\frac{d\rho}{dt} + \rho\nabla\cdot\vec{v} = 0 \quad , \tag{1}$$

$$\rho\frac{d\vec{v}}{dt} + \nabla p - \rho\nabla\Phi = 0 \quad , \tag{2}$$

$$\frac{dp}{dt} + \gamma p\nabla\cdot\vec{v} + (\gamma-1)\rho\mathcal{L} = 0 \quad , \tag{3}$$

$$\nabla^2\Phi + 4\pi G\rho = 0 \qquad (4)$$

Here, the unknown functions of space coordinates and time are: velocity -$\vec{v}$, density-ρ, pressure-p and gravitational potential per unit mass-Φ. Equation (3) is obtained from $dQ/dt + \mathcal{L} = 0$, by use of (1) and thermodynamical relation for dQ(the thermal energy per 1g). Here $\mathcal{L}$ is the gas heating-cooling function, defined as the difference between rate of cooling and rate of heating per gram. The system (1)-(4) should be completed by adding the algebraic equation for $\mathcal{L}$ and the ideal-gas-state equation.

Applying (1)-(4) to the gaseous component of the galactic disc one introduces the infinitesimally-thin-disc approximation and the cylindrical coordinates R, φ, z are used. For this purpose one introduces the surface density σ and the "surface" pressure π, defined as $f(R,\varphi,t) = \int_{-\infty}^{+\infty} f(R,\varphi,z,t)dz$. In addition, one requires that the temperature T and the velocity components v_R, v_φ are independent of z and also $v_z = 0$. Otherwise, Equations (1)-(4) are considered in the disc plane of symmetry (two degrees of freedom) where one has $\Phi = \Phi(R,\varphi,z,t)$ and the designation $\mathcal{L} = \mathcal{L}(\sigma, T)$ remains.

For the purpose of deriving of the perturbation equations, for each unknown function $f(R,\varphi,t)$ the form $f = f^0 + f'$ is introduced, as well as for Φ, and the basic state of the disc is defined as:

$$f^0 = f^0(R), \quad with \quad v_R^0 = 0\,, \mathcal{L}^0 = 0. \qquad (5)$$

By using (5) the system (1)-(4) in the plane is transformed into equations for perturbing functions f' and Φ' of an arbitrary type. In the case of small perturbations ($|f'/f^0| \sim \epsilon \ll 1$, and also for Φ'), in the linear approximation in ϵ it will be:

$$\frac{\partial\sigma'}{\partial t} + \Omega\frac{\partial\sigma'}{\partial\varphi} + \frac{1}{R}\left(\sigma^0\frac{\partial v'_\varphi}{\partial\varphi} + \frac{\partial R\sigma^0 v'_R}{\partial R}\right) = 0 \quad , (6)$$

$$\frac{\partial v'_R}{\partial t} + \Omega\frac{\partial v'_R}{\partial\varphi} - 2\Omega v'_\varphi + \frac{1}{\sigma^0}\left(\frac{\partial\pi'}{\partial R} - \frac{\sigma'}{\sigma^0}\cdot\frac{\partial\pi^0}{\partial R}\right) = \left(\frac{\partial\Phi'}{\partial R}\right)_{z=0} \qquad (7)$$

$$\frac{\partial v'_\varphi}{\partial t} + \Omega\frac{\partial v'_\varphi}{\partial\varphi} + \frac{\kappa^2}{2\Omega}v'_R = \frac{1}{R}\left[\left(\frac{\partial\Phi'}{\partial\varphi}\right)_{z=0} - \frac{1}{\sigma^0}\cdot\frac{\partial\pi'}{\partial\varphi}\right] \quad , \qquad (8)$$

$$\frac{\partial\pi'}{\partial t} + \Omega\frac{\partial\pi'}{\partial\varphi} + \frac{\partial\pi^0}{\partial R}v'_R + \frac{\gamma\pi^0}{R}\left(\frac{\partial Rv'_R}{\partial R} + \frac{\partial v'_\varphi}{\partial\varphi}\right) + (\gamma-1)\sigma^0\mathcal{L}' = 0 \quad , \qquad (9)$$

$$\nabla^2\Phi' + 4\pi G\sigma'\delta(z) = 0 \quad , \qquad (10)$$

with

$$\mathcal{L}' = \mathcal{L}_\sigma\sigma' + \mathcal{L}_T T'. \qquad (11)$$

Here

$$\kappa^2(R) = 4\Omega^2\left(1 + \frac{R}{2\Omega}\cdot\frac{d\Omega}{dR}\right)$$

is the square of the epicyclic frequency, $\Omega(R) = v_\varphi^0/R$ is the angular velocity of the disc material at a distance R from the galactic centre and $\delta(z)$ is Dirac's δ-function. Equation (11) is obtained as the linear expansion of the function $\mathcal{L}$ about the basic state, whereas the coefficients $\mathcal{L}_\sigma, \mathcal{L}_T$ are the partial derivatives of $\mathcal{L}$ in σ and T, respectively.

3. THE AMPLITUDES OF SPIRAL PERTURBATIONS IN THE DENSITY AND VELOCITIES

In the case of a spiral-form perturbation with m arms,

$$f' = \tilde{f}(R)e^{i(\omega t - m\varphi)},$$

Equations (6)-(9) become

$$i(\omega - m\Omega)\tilde{\sigma} - i\frac{m\sigma^0}{R}\tilde{v}_\varphi + \frac{1}{R}\cdot\frac{d}{dR}(\sigma^0 R\tilde{v}_R) = 0 \quad , (12)$$

$$i(\omega - m\Omega)\tilde{v}_R - 2\Omega\tilde{v}_\varphi + \frac{1}{\sigma^0}\left(\frac{d\tilde{\pi}}{dR} - \frac{\tilde{\sigma}}{\sigma^0}\cdot\frac{d\pi^0}{dR}\right) = \frac{d\tilde{\Phi}}{dR}, \qquad (13)$$

$$\frac{\kappa^2}{2\Omega}\tilde{v}_R + i(\omega - m\Omega)\tilde{v}_\varphi = -\frac{im}{R}\left(\tilde{\Phi} - \frac{\tilde{\pi}}{\sigma^0}\right), \qquad (14)$$

$$\left[i(\omega - m\Omega) + \frac{(\gamma-1)\mu\mathcal{L}_T}{\mathcal{R}_g}\right]\tilde{\pi} + (\gamma-1)(\sigma^0\mathcal{L}_\sigma - T^0\mathcal{L}_T)\tilde{\sigma} + \frac{\gamma\pi^0}{R}\left(\frac{dR\tilde{v}_R}{dR} - im\tilde{v}_\varphi\right) + \frac{d\pi^0}{dR}\tilde{v}_R = 0. \qquad (15)$$

The last equation is obtained from (9) and (11), and T' is eliminated by use of linear expansion in the ideal-gas equation, $\tilde{\pi} = \mathcal{R}_g \sigma T/\mu$, about the basic state: $\mathcal{R}_g$ is the gas constant, $\mu = const$-the mean molecular weight.

Now, using

$$\tilde{f}(R) = \hat{f}(R) e^{i\int k(R)dR},$$

one considers m-spiral (usually $m = 2$) slightly deviating from a circle: $|m/(kR)| \ll 1$. Also, $f^0(R)$ and the amplitudes $\hat{f}(R)$ will be considered as slowly varying functions, and $\hat{v}_\varphi \sim \hat{v}_R$. With these approximations Equations (12)-(15) are additionally simplified. In a further step $\tilde{\pi}$ will be eliminated from (13) through (15) and (12), and new variables appear

$$\overline{\nu} = \frac{\omega - m\Omega}{\kappa},$$

$$k_T = \frac{(\gamma-1)\mu\mathcal{L}_T}{\mathcal{R}_g c_s}, \quad k_\sigma = \frac{(\gamma-1)\mu\sigma^0\mathcal{L}_\sigma}{\mathcal{R}_g T^0 c_s}, \qquad (16)$$

where $c_s = (\partial\pi^0/\partial\sigma^0)^{1/2}$ is the adiabatic sound speed in the disc plane. Equations (12)-(14) become

$$i\overline{\nu}\hat{\sigma} + i\frac{k\sigma^0}{\kappa}\hat{v}_R = 0, \qquad (17)$$

$$i\frac{k}{\sigma^0\kappa}c_s^2 h\hat{\sigma} + i\overline{\nu}\hat{v}_R - \frac{2\Omega}{\kappa}\hat{v}_\varphi = i\frac{k\hat{\Phi}}{\kappa}, \qquad (18)$$

$$\frac{\kappa}{2\Omega}\hat{v}_R + i\overline{\nu}\hat{v}_\varphi = 0, \qquad (19)$$

with

$$h = \frac{\frac{1}{\gamma}(x_T - x_\sigma) + i\overline{\nu}}{x_T + i\overline{\nu}},$$

$$x_T = \frac{k_T c_s}{\kappa}, \quad x_\sigma = \frac{k_\sigma c_s)}{\kappa}. \qquad (20)$$

For a given amplitude of the gravitational-potential perturbation the solution of the system (17)-(19) for the unknowns $\hat{\sigma}, \hat{v}_R, \hat{v}_\varphi$ is

$$\hat{\sigma} = \sigma_0\frac{k^2\hat{\Phi}}{\kappa^2}F_\sigma, \quad \hat{v}_R = -\frac{k\hat{\Phi}}{\kappa}F_{v_R}, \quad \hat{v}_\varphi = -i\frac{k\hat{\Phi}}{2\Omega}F_{v_\varphi}, \qquad (21)$$

where

$$F_\sigma = F_{v_\varphi} = \frac{1}{1 - \overline{\nu}^2 + x^2 h},$$

$$F_{v_R} = \overline{\nu}F_\sigma, \quad x^2 = \frac{k^2 c_s^2}{\kappa^2}. \qquad (22)$$

As well known, in the case of a given $k(R) \in Re$ with no thermal processes it is $\omega \in Re$, i.e. $\overline{\nu} \in Re$. In a general case it is $\overline{\nu} = u + iw$ where

$$u = \frac{Re(\omega) - m\Omega}{\kappa}, \quad w = \frac{Im(\omega)}{\kappa}.$$

Since $\mathcal{L} = 0$ causes $w = 0$, from (20) and (22) it follows

$$h = 1, \quad F_\sigma \equiv F_\sigma^0 = \frac{1}{1 - u^2 + x^2},$$

so that (21) appears as the well-known result concerning the response of the gaseous disc to a given gravitational-potential perturbation (e.g. Rohlfs, 1977). If $\mathcal{L} \neq 0$, when $|w/u| \ll 1$ in a region remote from the corotation, the linear approximation yields

$$h = 1 + i\alpha, \quad F_\sigma = F_\sigma^0(1 - i\alpha x^2 F_\sigma^0), \qquad (23)$$

with

$$|\alpha| = |\frac{(\gamma-1)x_T + x_\sigma}{\gamma u}| \ll 1.$$

If the amplitudes $\hat{q} \equiv (\hat{\sigma}, \hat{v}_R, \hat{v}_\varphi)$ in the absence of thermal processes are denoted as $\hat{q}(0)$, the results (21) can be written through (23) in the form

$$\frac{\hat{\sigma}}{\hat{\sigma}(0)} = \frac{\hat{v}_\varphi}{\hat{v}_\varphi(0)} = 1 - i\beta_1,$$

$$\frac{\hat{v}_R}{\hat{v}_R(0)} = 1 - i\beta_2, \qquad (24)$$

with $|\beta_2| \sim |\beta_1| \ll 1$, where

$$\beta_1 = \alpha x^2 F_\sigma^0, \quad \beta_2 = \beta_1 + \frac{w}{u}.$$

The solutions (24) for the amplitudes $\hat{q}(R)$ of spiral waves $q = \hat{q}\{exp[i(\omega t - m\varphi + \int kdR)]\}$ determine the linear response of the Galaxy's gaseous disc to a gravitational-field perturbation with the presence of the heating-cooling processes.

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ТОПЛОТНИ ПРОЦЕСИ И СПИРАЛНИ ТАЛАСИ ГУСТИНЕ I. РЕАКЦИЈА ГАЛАКТИЧКОГ ДИСКА НА ПОРЕМЕЋАЈ ГРАВИТАЦИОНОГ ПОТЕНЦИЈАЛА

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Оригинални научни рад

Разматра се гасовити диск Галаксије у диференцијалној ротацији, са процесима загревања и хлађења. У апроксимацији бесконачно танког диска изводе се линеарне једначине малих поремећаја. Дају се решења за амплитудне функције спиралних таласа густине и брзина.