

MUTUAL PERTURBING EFFECTS OF MINOR PLANETS IN THEIR ORBITAL ELEMENTS

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SUMMARY: Mutual perturbing effects in orbital elements are computed for 8 chosen pairs of minor planets with mutual close encounters. For two pairs, the resp. perturbing masses are known, whereas for the remaining pairs the computation has been performed with assumed masses. It has been obtained that orbital elements of some perturbed minor planets show measurable perturbations.

1. INTRODUCTION

The cinemactical data for minor planets we dispose with, allow a more or less good knowledge of their motion. However, in general, the gravitational influences of these small bodies are negligible, and so are the possibilities of the dynamical determination of their masses. Nevertheless, some possibilities exist for large minor planets, offered by the study of their perturbing effects on other minor planets on the occasion of close encounters. The repetition of close encounters of a pair of minor planets, with similar geometrical characteristics (leading to a cumulative of gravitational effects), can be only temporary (temporary resonance). On the bases of such temporary resonances the masses of three largest minor planets have been determined (Schubart and Matson, 1979; Landgraff, 1988). For other minor planets the prospect in that sense is practically impossible, so that attempt at the determination of masses must be looked for among single close en-

counters. From such a single encounter of the minor planet 829 Academia with 10 Hygiea the mass of the latter has been determined (Scholl et al., 1987). The question is which gravitational effects could be expected for other mutual close encounters of minor planets. Lazović and Kuzmanoski (1979, 1980, 1981, 1983) have studied simulated very close encounters of minor planets, assuming them at the minimal distances of their respective orbits. They have shown that important perturbing effects of their orbital elements could have been expected, even in the cases when the period of mutual gravitational influence was short. For such short-time perturbations, Simovljevič (1979a, 1979b, 1979c, 1979d, 1981a, 1981b) has given simple and expeditive methods of calculation.

In this paper, the gravitation influences of one minor planet on the other have been computed for 8 pairs, with a purpose of the determination of perturbing effects between minor planets in cases of real close encounters.

Table 1.

<i>j</i>	<i>k</i>	Epoch of elements		ρ	Moment of close enc.		I
		D_j	JD		JD	V_{rel}	
2	3131	523	2446040.5	0.01191	2446038.6	10.85	36.3
4	413	501	2448435.5	0.01058	2448433.3	8.21	11.6
3	1767	244	2445335.5	0.00546	2445334.0	4.54	5.4
19	3486	215	2450215.5	0.00211	2450217.1	2.30	4.7
45	673	214	2448115.5	0.00338	2448116.7	2.55	6.7
65	526	245	2445875.5	0.00593	2445876.0	3.33	1.6
81	767	124	2446095.5	0.00060	2446095.9	4.57	7.7
92	2950	132	2446405.5	0.00261	2446402.9	3.09	1.7

Table 2.

		E_{1988}	E_{1991}	E_{1990}^1	E_{1990}^2	ΔE
45	M_0	101.13367	3.42720	257.65908	257.65087	.00821
	ω	85.77431	86.93549	86.25222	86.25976	-.00754
	Ω	147.48167	147.35530	147.35807	147.35814	-.00007
	i	6.60164	6.61077	6.60720	6.60723	-.00003
	e	.0829974	.0802935	.0807798	.0807717	.0000081
	μ	.21978486	.21963320	.21930955	.21930921	.00000034
	a	2.7193860	2.7206378	2.7233138	2.7233166	-.0000028
673	M_0	218.36780	122.10541	16.15671	15.51474	.64197
	ω	248.03528	234.77229	239.45968	240.10144	-.64176
	Ω	226.66684	226.50899	226.58852	226.58862	-.00010
	i	2.87648	2.87382	2.87267	2.87277	-.00010
	e	.0120241	.0122297	.0126038	.0126812	-.0000774
	μ	.20885218	.20880489	.20844963	.20844980	-.00000017
	a	2.8134770	2.8139018	2.8170980	2.8170965	.0000015

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Table 3.

j	k	m_j (in solar mass)	$\Delta\mu$	$\Delta\varphi$	$\Delta\Omega$	Δi	$\Delta\omega$	ΔM_0	
2	3131	$1.1 \cdot 10^{-10}$	-0.00	-0.02	-0.07	.00	.23	.24	
			-0.03	.01	.13	.01	.25	.26	
4	413	$1.2 \cdot 10^{-10}$.00	.00	-0.00	.02	.03	.04	
			.01	.01	-0.00	.02	.03	.01	
3	1767	$1.0 \cdot 10^{-11}$.00	.00	.03	.00	.03	.02	
			.00	.00	.04	.00	.04	.03	
			$5.0 \cdot 10^{-12}$.00	.01	.00	.01	.01	.01
19	3486	$1.0 \cdot 10^{-11}$.00	.04	-0.00	.00	.04	.09	
			.00	.06	.03	.02	.07	.05	
			$5.0 \cdot 10^{-12}$.00	.02	-0.00	.00	.02	.04
			.00	.03	.01	.01	.04	.03	
45	673	$1.0 \cdot 10^{-11}$.00	.01	.04	.00	1.18	1.15	
			.00	.00	-0.26	.00	1.58	1.54	
			$5.0 \cdot 10^{-12}$.00	.00	.02	.00	.59	.57
			.00	.00	-0.13	.00	.79	.77	
65	526	$1.0 \cdot 10^{-11}$.00	.00	-0.02	.00	.07	.09	
			.00	.01	-0.02	.00	.12	.12	
			$5.0 \cdot 10^{-12}$.00	.00	-0.01	.00	.04	.04
			.00	.01	-0.01	.00	.06	.06	
81	767	$1.0 \cdot 10^{-11}$.00	.04	.97	.00	.03	.02	
			.00	.04	1.14	.01	.27	.19	
			$5.0 \cdot 10^{-12}$.00	.02	.48	.00	.01	.01
			.00	.02	.57	.00	.13	.09	
92	2950	$1.0 \cdot 10^{-11}$.00	.03	.07	.00	-.03	.02	
			.00	.03	.10	.00	-.09	.12	
			$5.0 \cdot 10^{-12}$.00	.01	.03	.00	.01	.01
			.00	.01	.05	.00	.05	.06	

2. RESULTS AND CONCLUSIONS

The pairs of minor planets with close encounters, for which the perturbing effects have been computed, were taken from Kuzmanoski's list (1992) having in mind minimal close encounter distances, relative velocities and diameters. Besides, we have included the pairs (2,3131) and (4,413) because of known masses of 2 Pallas and 4 Vesta, despite the fact that their closest distances slightly exceed 0.01 AU and their relative velocities are great. On Table 1. the considered pairs are given; the first (large) minor planet being the perturbing, the second one perturbed, the diameter D_j of the perturbing one, the

epoch of orbital elements, the mutual distance ρ at the moment of closest encounter, that moment t , the relative velocity and the mutual inclination of orbital planes. The pair (45,673) is particularly interesting as its close encounter happened in 1990. As the calculus of the perturbation resulting from the action of major planets, when looking for close encounters, has been performed with the osculatory elements for the epoch JD 2447400.5, whereas we dispose now with orbital elements for JD 2448600.5 (which include new observations), we have verified the corresponding changes of the results. On Table 2. the Elements E_{1988} and E_{1991} are given for these two osculating epochs, the elements E_{1990}^1 (computed with E_{1988}), E_{1990}^2 (computed with E_{1991}) and the residu-

als $\Delta E = E_{1990}^1 - E_{1990}^2$. Residuals are largest for the mean anomalies and for the argument of perihelion ω_{673} , which are the consequence of the exceptionally fast changes of that element. The closest distance ρ increased from 0.00338 to 0.00368, for the moment JD 2448116.9 (0.2 days later).

The perturbations, resulting from the action of large minor planets, of the orbital elements of corresponding planets in pairs, are calculated by means of the Gauss-Encke method. For the first two pairs the perturbations have been calculated with known masses for 2 Pallas and 4 Vesta, whereas for other pairs it was done with two optimistically assumed perturbing masses. On Table 3. are given obtained results; the values of perturbations up to the moment of closest encounter in the first row, and their final values in the second one. There have been no perturbations in the mean daily motions, whereas the angles of eccentricity and the inclinations have shown either very small perturbations or not at all. The values of perturbations are greatest in the longitudes of perihelia and mean anomalies and, in general rule, with opposite signs. Perturbations in longitudes of ascending nodes have different values and the largest are for the pair (81,767), probably because that pair has the smallest close encounter distance. It can be noted that the perturbations resulting from the action of Pallas are greater than those originated by Vesta in spite of the fact that Vesta's mass is slightly greater, and the respective distance and relative velocity are smaller. Such cases can be expected for other close encounters as well, the perturbing effects depending on the geometric relationship of the orbits. In general, such changes in the orbital elements can provoke measurable effects, when expressed in the spherical coordinates of perturbed minor planets, of the same order as the perturbations of orbital elements themselves.

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МЕЋУСОБНИ ПОРЕМЕЋАЈНИ УТИЦАЈИ МАЛИХ ПЛАНЕТА У ЊИХОВИМ ПУТАЊСКИМ ЕЛЕМЕНТИМА

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За 8 одабраних парова малих планета са међусобним блиским прилазима, израчунати су међусобни поремећајни ефекти у путањским елементима. За два пара масе поремећајних малих план-

ета су познате, а за преостале парове рачун је изведен са претпостављеним масама. Показано је да долази до мерљивих промена у неким путањским елементима поремећених малих планета.