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Review paper

MAGNETIC FIELDS IN COOL STARS

M. Karabin

Astronomical Institute, Faculty of Mathematics, Studentski trg 16, 11000 Belgrade, Yugoslavia

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SUMMARY: Our present understanding about the origin and structure of magnetic fields in cool stars is described in this review. Fundamental equations for α-ω dynamo theory and magnetic "flux tubes", together with structure of the photospheric and atmospheric magnetic fields are presented.

1. INTRODUCTION

Stellar magnetic fields have provoked more attention since it was realized that all irregularities and activity of the Sun are manifestation of its magnetic fields. The Sun is the only star whose magnetic field and surface motions (differential rotation, convection) can be resolved over the disk. For all other stars measurements are restricted to the integrated Zeeman effect which can be detected only if the algebraic mean in the line-of-sight component exceeds 10^2 gauss. Such direct measurements of magnetic fields for the late-type stars are very rear (Saar and Linsky, 1985). It has been inferred from the theory and observations of stellar activity, that the cool stars with convective envelopes are magnetic in the same way as the Sun is magnetic.

We are going to give here a brief review of our present understanding about the origin and structure of magnetic fields in cool stars.

2. FLUX TUBES AND THE SOLAR DYNAMO

Most solar physicists have accepted the supposition that the solar magnetic field is generated and maintained by dynamo mechanism in the convection zone. There are very few authors who reject the dynamo theory (Piddington, 1978; Layzer et al., 1979) replacing it by primordial field, but not in a convicing way. The solar dynamo theory has been built on convection and differential rotation by the use of MHD-approximation. More then 35 years have passed since the early work of Parker (1955), when general ideas and equations for the solar dynamo were established. During this period of time many scientist made great efforts to modify and improve the theory in order to explain diversity of observational results such as: magnetic reversals with opposit polarities in the north and south hemispheres, the migration of sunspots and active regions towards the equator with time, the appearance and behavior of different magnetic structure etc.

We are well aware of the risk to mention only several among so many excellent and important works of numerous authors. Nevertheless, we think that contributions done by Leighton (1964, 1969), Frisch et al. (1975), Krause (1976), Uno and Ribes (1976) and Parker (1977, 1978, 1979, 1981, 1985) together with those authors who will be mentioned further on in this review, made α-ω dynamo theory a successful one. Fundamental equations for standard linear α-ω dynamo are:

Vector equation of momentum:

$$\rho\frac{d\vec{v}}{dt} = -\nabla p + \rho\vec{g} + \frac{1}{c}\vec{j}\times\vec{B}, \quad (1)$$

where ρ is the gas density, p is the gas pressure, $\vec{g}$ the gravitational acceleration, $\vec{j}$ the electric current density, and $\vec{B}$ the magnetic field.

Ohm's law:

$$\vec{j} = \sigma(\vec{E} + \frac{1}{c}\vec{v}\times\vec{B}), \quad (2)$$

where σ is the scalar electrical conductivity, $\vec{v}$ the plasma velocity. The electrodynamic quantities $\vec{j}$, $\vec{E}$ and $\vec{B}$ are determined by Maxwell's equations:

$$\begin{aligned} \nabla\times\vec{B} &= \frac{4\pi}{c}\vec{j} \\ \nabla\cdot\vec{B} &= 0 \\ \nabla\times\vec{E} &= -\frac{1}{c}\frac{\partial\vec{B}}{\partial t} \\ \nabla\cdot\vec{E} &= 4\pi\rho_e \end{aligned} \quad (3)$$

(ρ_e is the charge density).

Using relations (1), (2) and (3) the familiar equations was obtained:

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}) + \eta\nabla^2\vec{B}, \quad (4)$$

where the magnetic diffusivity η is given by:

$$\eta = \frac{c^2}{4\pi\sigma}. \quad (5)$$

The ratio of two terms on the right-hand side of (4) is determined by the magnetic Reynolds number:

$$R_m = \frac{vL}{\eta}, \quad (6)$$

where L is the characteristic length.

In most of astrophysical problems due to large L, $R_m \gg 1$, magnetic fields are "frozen into the plasma" and there is no relative motion between the magnetic field and plasma in direction $\vec{n}$ perperdicular to the field $\vec{B}$. Local changes of the magnetic field are determined by convection:

$$\frac{\partial\vec{B}}{\partial t} = \nabla\times(\vec{v}\times\vec{B}).$$

The flow concentrates field lines in "flux tubes":

$$\frac{4\pi}{c}\vec{j}\times\vec{B} = (\nabla\times\vec{B})\times\vec{B}. \quad (7)$$

If the field is strong enough, it can halt the flow locally. An estimation of the field strength may be obtained from the kinetic energy density:

$$B_{eq}^2 = 4\pi\rho v^2. \quad (8)$$

B_{eq} is the equipartition field.

The magnetic field and convection coexist beside one another. The field is concentrated in the flux tubes while most of the plasma is convective and field-free. In such a medium mechanical equilibrium of a flux tube is given by:

$$-\nabla p + \rho\vec{g} + \frac{c}{4\pi}(\vec{B}\nabla)\vec{B} - \nabla\frac{B^2}{8\pi} = 0 \quad (9)$$

Magnetic pressure obtained from (9) has two terms: the isotropic one $\left(\frac{B^2}{8\pi}\right)$ and the tension along the lines of force $\left(\frac{-B^2}{4\pi}\right)$:

$$p_m = \frac{B^2}{8\pi} - \frac{B^2}{4\pi}. \quad (10)$$

According to $R_m \gg 1$ in the field direction $\vec{l}$ (9) becomes:

$$-\frac{\partial p}{\partial l} + \rho(\vec{g}\cdot\vec{l}) = 0, \quad (11)$$

while in direction $\vec{n}$ (see Fig. 1):

$$\frac{\partial}{\partial n}(p + \frac{B^2}{8\pi}) = 0. \quad (12)$$

In the flux tube:

$$p_i + \frac{B^2}{8\pi} = Const. \quad (13)$$

Consequently the internal (p_i) and the external (p_e) pressure are related by:

$$p_i + \frac{B^2}{8\pi} = p_e. \quad (14)$$

As reduced internal pressure implies reduced density, the buoyancy force, F_b, which acts on a flux tube, has this form:

$$F_b = Sg(\rho_e - \rho_i), \quad (15)$$

where S is the cross-section of the tube, $(\rho_e - \rho_i)$ is the difference between the external and internal densities.

The buoyancy force should rise flux tubes approximately by the Alfvén speed. Magnetic flux emerges at the photopshere as a bundle of rising flux loops (Zwaan, 1985). The radiative loss from the loop is not balanced by convective heat, because convection is reduced by the magnetic field. Therefore, gas in the flux loop tends to cool and slump starting a convective downflow. As temperature decreases the gas pressure decreases too, and the flux tube is compressed until the increased magnetic pressure, $\frac{B^2}{8\pi}$, balances the gas pressure difference (14).

The magnetic field becomes concentrated in the thin flux tubes as a result of convective instability. This process named "convective collaps" creates flux tubes of the order of 10^3 gauss. The theory of formation of such thin and strong flux tubes has some weak points (Piddington 1978, Parker 1979, Spruit 1981) which have not been overcome till now (Zwaan and Cram, 1989). The assumed small cross section of the flux tube (~ 200km) is still below the instrumental resolution therefore for the observational proof we have to wait. At present we are able to detect only the bundle of the flux tubes at the photospheric level.

All solar dynamo theories are based on the action of differential rotation upon general "poloidal" magnetic field in the meridional plane. The shears in rotation should induce a "toroidal" field about the axis of rotation. Then, lifting plasma motions (convection) are twisted due to Coriolis forces producing "cyclonic convective cells". These cells are formed on the toroidal field having components which create a new poloidal field with the direction opposit to the previous one. For the detailed mathematical analysis of the dynamo in turbulent fluids see Parker 1979 p. 532.

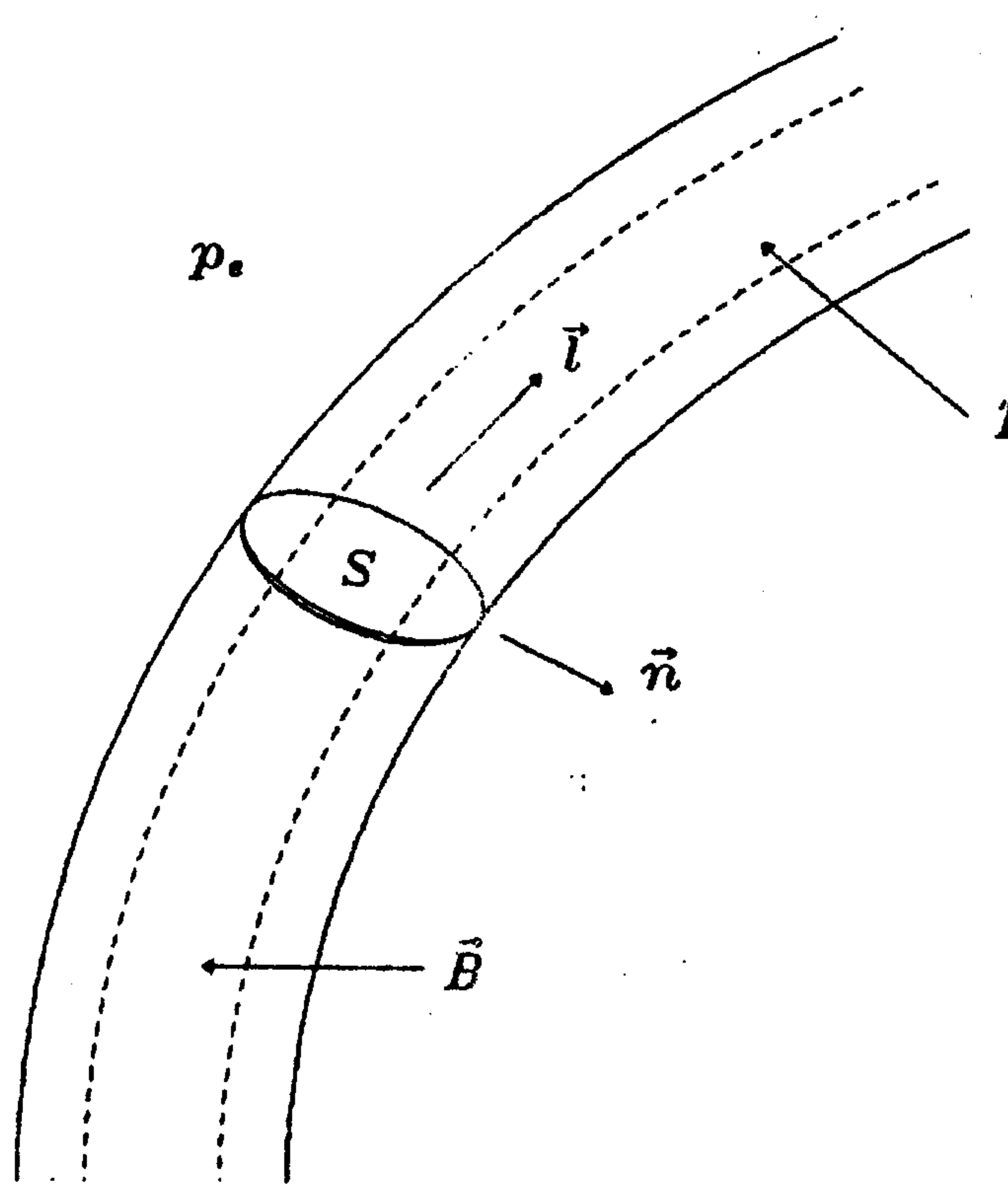

Fig. 1. **Magnetic flux tube.**

The important parameter in the dynamo theory is the Rossby number:

$$R = \frac{u_c}{L\omega}, \tag{16}$$

where u_c is the characteristic convective velocity, L is the characteristic convective length, and ω is the angular velocity in the convection zone.

The regeneration process of the poloidal field is named "α-effect". The parameter α represents "the cyclonic turbulence" (i.e. the mean rotational speed of eddies). The regeneration will take place only if the Rossby number is:

$$R < 1. \tag{17}$$

The mechanism mentioned above is too complex to be presented by a general mathematical theory. Such a theory does not exist for the time being. The appropriate solution is found by introducing the hydrodynamic turbulence which can be represented by the therm:

$$\alpha\overline{\vec{B}}, \tag{18}$$

modifying the equation (4) to the form:

$$\frac{\partial\overline{\vec{B}}}{\partial t} = \nabla \times (\overline{\vec{v}} \times \overline{\vec{B}} + \alpha\overline{\vec{B}}) + \eta_t\nabla^2\overline{\vec{B}}, \tag{19}$$

in which $\overline{\vec{B}}$ is the mean magnetic field, and $\overline{\vec{v}}$ is the mean plasma velocity.

Several authors searched for solutions of the equation (19): Yoshimura (1975), Stix (1976), Gilman and Miller (1981), Glatzmaier and Gilman (1981). The problem is that mean-field dynamo theory requires increase of rotation rate $\frac{\partial\omega}{\partial r} > 0$ with depth, what is not confirmed till now. There is the indication of an opposite result $\frac{\partial\omega}{\partial r} < 0$ (Duvall et al., 1986).

Nevertheless, it is believed that α-ω dynamo is a promising theory.

3. THE PHOTOSPHERIC MAGNETIC FIELD

Over the surface of the Sun the magnetic field is inhomogenious with pronounced tendency of self-concentration into isolated flux tubes of 1500 gauss or more. In spite of that, general photospheric field of the Sun as a star is very weak: 0.15 gauss (Severny et al., 1970). Such field would not be detectable if the Sun were not so close to us. Explanation for so weak general field is small "filling factor" (area occupied by the magnetic field). Namely, large fractions of photosphere are essentially field-free whereas the concentrated magnetic flux covers <10% of the surface only.

Concentration of magnetic flux (according to present resolve possibilities) appears in the folowing hierarchy of magnetic elements: faculae, network and knots ($0.5B^* \leq B \leq B^*$; $1.5H_p \leq R \leq 2H_p$), pores ($1.1B^* \leq B \leq 1.3B^*$; $3.5H_p \leq R \leq 13H_p$), spots ($1.4B^* \leq B_u \leq 1.7B^*$; $R \geq 13H_p$). Notation:

R – photospheric radius at $\tau_5 = 1$;
H_p – pressure scale height;
B – magnetic field strength;
B^* – characteristic field strength ($B^* > B_{eq}$);
B_u – umbral field strength.

(All data are from Zwaan and Cram, 1989).

Magnetically active cool stars indicate the existence of strong surface fields (Robinson et al., 1980) with significant "filling factor" (20–45%) (Wilson, 1978; Fossat, 1984; Noyes, 1986). It may be an indicator for higher activity level of younger stars (Vukićević-Karabin and Arsenijević, 1986). Majority of these cool stars are late dwarfs. So, for the time of $\sim 10^9$ years the Sun reaches its middle age whereas these dwarfs are still young. According to §2 we can conclude that, probably, every cool star (F5 – M) has a magnetic field generated by dynamo process and concentrated in flux tubes. Because of the bouyancy force (15), they emerge at photospheres. In the areas of magnetic structures the photospheric gas cools and descends the result being further concentration of the magnetic field.

Although it is assumed that magnetic field formation in cool stars is similar to that of the solar field, these fields do not necessarily have identical structure. This is the reason why we use the expression "filling factor" for cool stars. It is found that the "filling factor" varies with time (cycles of activity) but with the exeption of very high values (45%) it accounts usually for about 10–20%. This means that most of photospheric areas are field free. The magnetic field does not control the transport of energy from photospheres of cool stars. This is not the case in the stellar atmosferes, especially in the coronae as we shall se.

4. THE ATMOSPHERIC MAGNETIC FIELD

The structure of magnetic field in the stellar atmospheres is determined by the exponential decrease of the gas pressure with height. Magnetic elements concentrated in small areas of the photosphere fan upwards with increasing height. The "filling factor" increases with height untill merge of flux tubes. The merging height according to Spruit (1983) is:

$$h_m \approx 2H_p \ln(f_p^{-1}), \qquad (20)$$

where H_p is the pressure scale height, f_p is photospheric "filling factor".

The merging height marks the separation of *photospheric regime* (isolated flux tubes) from *atmospheric regime* where plasma is everywhere magnetized (Fig. 2). Only open field lines are extended into interplanetary space, whereas closed field lines forming coronal loops are rooted in flux tubes embedded in the photosphere and convection zone.

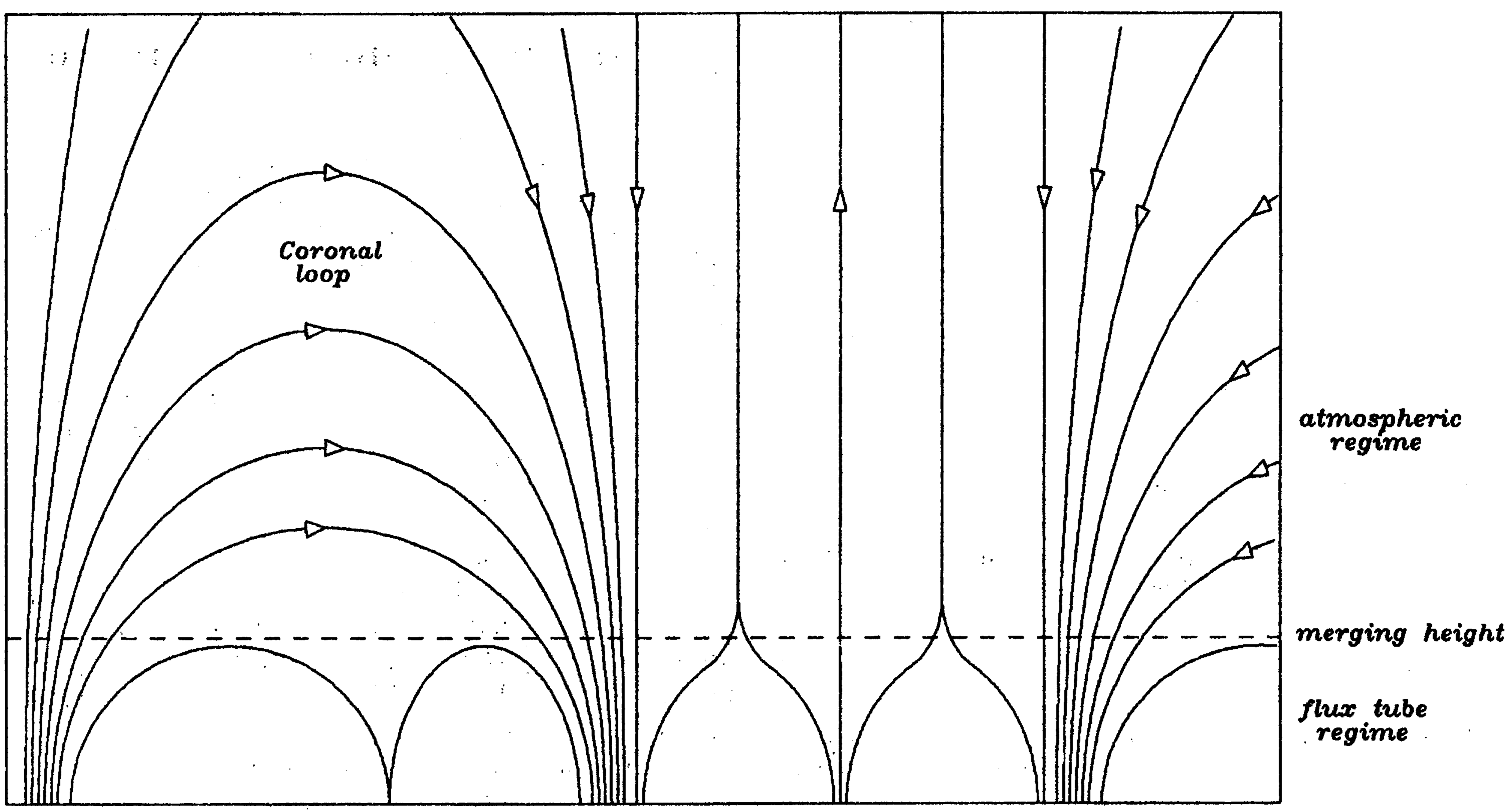

Fig. 2. "Open" and "closed" magnetic field (according to Zwaan and Cram, 1989).

The upper corona and interplanetary space are everywhere filled by open field lines. These lines are in direct contact. In lower atmosphere magnetic field structure becomes increasingly complicated. There are two topologically distinct types of closed field lines. One is characterised by a dome-like surface and the other is characterized by a separator on the top (Fig. 2). In closed field structure loops are thermally isolated (Priest, 1984), and plasma in them can be very warm ($T \geq 10^6$K). In open field structure stellar wind flows to interstellar space parallel to the field lines. According to $R_m \gg 1$ and (11):

$$\vec{j} \parallel \vec{B}. \tag{21}$$

From equations (10) and (14) the ratio of gas to magnetic pressure is:

$$\beta = \frac{8\pi p}{B^2}. \tag{22}$$

In the photosphere inside the flux tubes $\beta \approx 1$ and outside the tubes $\beta \gg 1$. In the atmosphere and especially in the corona $\beta \ll 1$. It means that in the atmosphere the kinetic energy density is small compared to the magnetic energy density, consequently the magnetic field controls the processes and structure of the atmosphere. As strength and distribution of the magnetic field in the atmosphere cannot be measured directly, the emission of some spectral lines (CaII H and K, MgII h and k etc.) serves for this purpose (Howard, 1959; Bumba and Topolova, 1967; Mewe et al., 1981). The Skylab mission (1973) provided observations in soft X-rays, which gave us a picture of "open" and "closed" magnetic structures in the solar corona. There are observations in X-rays for cool stars from the Einstein Observatory (Aires et al., 1981; Linsky, 1981; Golub, 1983). It is believed that loop-like structures are important in the atmospheres of many cool stars.

We may conclude that magnetic fields play a significant role in the physical processes in various parts of any cool star (including heating which will be analyzed elsewhere).

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МАГНЕТНА ПОЉА ХЛАДНИХ ЗВЕЗДА

М. Карабин

Институт за астрономију, Студентски трг 16, Београд, Југославија

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Прегледни чланак

У овом прегледном раду изложено је данашње схватање порекла и структуре магнетних поља хладних звезда. Дате су фундаменталне једначине за $\alpha - \omega$ динамо теорију и "магнетне цеви" и приказана је структура фотосферских и атмосферских магнетних поља.